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The complete $AdS_4 \times CP^3$ superspace for the type IIA superstring and D-branes

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Abstract: We lift the bosonic $AdS_4 \times CP^3$ solution of type IIA supergravity preserving 24 supersymmetries to a $D = 10$ superspace which has 32 Grassmann-odd directions. The type IIA superspace is obtained from $D = 11$ via dimensional reduction of the coset superspace OSp(8|4)/SO(7) × SO(1, 3) by realizing the latter as a Hopf fibration over the former. This construction generalizes to superspace the Hopf fibration of S^7 as a U(1) bundle over CP^3, and is suitable for writing the explicit form of Green-Schwarz-type actions encoding the dynamics of the type IIA string and branes in the $AdS_4 \times CP^3$ superbackground. We show that the OSp(6|4)/U(3) × SO(1, 3) supercoset string action describes only a subsector of the complete Green-Schwarz superstring. Thus, even though the superstring equations of motion in the OSp(6|4)/U(3)×SO(1, 3) subsector are classically integrable, the fact that the full $AdS_4 \times CP^3$ superspace is not a supercoset requires the use of more general methods to determine whether the superstring in the complete $AdS_4 \times CP^3$ superbackground is classically integrable.

Keywords: Superstrings and Heterotic Strings, D-branes, AdS-CFT Correspondence, Supergravity Models

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Contents

1 Introduction

Recent progress in understanding the AdS_4/CFT_3 correspondence has been triggered by the construction of Bagger-Lambert-Gustavsson-type models based on tri-algebras [1, 2, 4] and by the model due to Aharony, Bergman, Jafferis and Maldacena (ABJM) [5].[1] These new models — based on 3-dimensional $\mathcal{N}$-extended superconformal Chern-Simons gauge

[1] The ABJM Lagrangian is a special case of the $\mathcal{N} = 4$ superconformal Chern-Simons theories written down in [6].

JHEP03(2009)015

theories coupled to scalar supermultiplets — have been conjectured to provide an effective low energy description of multiple coincident M2-branes in M-theory, with the ABJM theory at level k describing the physics of multiple M2-branes on an R^8/Z_k orbifold [5]. These novel three dimensional theories provide us with new tools for studying the AdS_4/CFT_3 duality from the boundary field theory point of view, and may shed new light on the landscape of AdS_4 vacua in string theory.

The $\mathcal{N} = 6$ Chern-Simons theory with gauge group $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ constructed in [5] describes M-theory on $AdS_4 \times S^7/Z_k$. There is a region in the parameter space of the ABJM theory[2] where the bulk description is given in terms of perturbative type IIA string theory on the $AdS_4 \times CP^3$ background, which preserves 24 out of 32 supersymmetries. Therefore, in order to study this new type of holographic correspondence using the bulk description, one needs an explicit form of the superstring action on the type IIA superspace whose bosonic body is $AdS_4 \times CP^3$. Likewise, writing down the action of D-branes on the $AdS_4 \times CP^3$ superbackground is useful, as D-branes in $AdS_4 \times CP^3$ play an important role in the duality, since they describe various local and non-local operators in the dual gauge theory [5, 7–11]. Of course, the Green-Schwarz-type form of the superstring action and superbrane actions in generic superbackgrounds are well known [12–19]. The challenge is to obtain the explicit form of the superstring and superbrane actions[3] for the various AdS_4/CFT_3 superbackgrounds, by finding the explicit dependence of the supervielbeins, NS-NS and RR superfields on the 32 fermionic coordinates of the type IIA superbackground of interest.

Analogous demand for explicit actions for the superstring and branes arose in the early studies of the AdS_5/CFT_4 and AdS_4/CFT_3 correspondence. In the maximally supersymmetric $AdS_5 \times S^5$ superbackground, the supergeometry is described by the coset superspace $\mathrm{SU}(2,2|4)/\mathrm{SO}(5) \times \mathrm{SO}(1,4)$, and the explicit form of the action for the type IIB superstring was found in [21, 22] while the D3-brane action was constructed in [23]. Analogous actions were derived for the M2-brane [24] and the M5-brane [25, 26] in the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ superbackgrounds respectively, which are described by the supercosets $\mathrm{OSp}(8|4)/\mathrm{SO}(7) \times \mathrm{SO}(1,3)$ and $\mathrm{OSp}(6,2|4)/\mathrm{SO}(4) \times \mathrm{SO}(1,6)$.

The construction of the superstring and brane actions in the $AdS_4 \times CP^3$ background is significantly more complicated, as the background preserves only 24 out of the 32 supersymmetries of type IIA supergravity. A coset superspace whose isometries are those of the $AdS_4 \times CP^3$ vacuum is $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$. Its bosonic body is the desired $AdS_4 \times CP^3$ geometry and its Grassmann-odd subspace is 24-dimensional. Therefore, $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ is a particular solution of the type IIA supergravity constraints which can be regarded as a submanifold in the general $AdS_4 \times CP^3$ IIA superspace, whose Grassmann-odd sector is 32-dimensional.

A sigma-model action for the superstring propagating in the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ submanifold of the complete type IIA superspace was constructed and analyzed in [27–31]. This action can be regarded as the Green-Schwarz action for the superstring in an $AdS_4 \times$

[2]Corresponding to $N^2 \gg \lambda^{5/2}$, where λ is the 't Hooft coupling of the ABJM theory.

[3]The superstring action to quadratic order in the fermionic coordinates is known in an arbitrary superbackground [20].

JHEP03(2009)015

CP^3 superspace with 32 fermionic directions in which the 16-parameter kappa-symmetry has been partially fixed in order to eliminate the 8 fermionic coordinates of the string corresponding to the 8 broken supersymmetries. With this interpretation, only 24 fermionic modes on the string worldsheet remain and these are described by the sigma-model based on the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ supercoset. This fixing of kappa-symmetry restricts the motion of the string to a submanifold of bosonic dimension 10 and fermionic dimension 24 in the total type IIA superspace. As already noted in [27], the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ sigma-model action does not describe all possible motions of the string in the $AdS_4 \times CP^3$ superspace. In particular, if the string moves entirely in AdS_4, the number of kappa-symmetries of this sigma-model gets increased from 8 to 12. This indicates that this dynamical sector of the theory cannot be attained from the gauge choice for fixing kappa-symmetry of the Green-Schwarz string action that yields the coset superspace. In this sector of the theory, four of the modes associated with the eight broken supersymmetries are dynamical fermionic degrees of freedom of the superstring. The reason behind this is that when the string moves entirely in AdS_4, its kappa-symmetry projector commutes with the projector which singles out the 8 broken supersymmetries, and therefore it cannot eliminate all the corresponding fermionic modes but only half of them.

Therefore, the study of the general classical and quantum motion of the superstring in $AdS_4 \times CP^3$ cannot be achieved using the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ supercoset. We need to find an action that includes the extra dynamical fermionic modes. On general grounds, this is given by the Green-Schwarz superstring action in the $AdS_4 \times CP^3$ superspace with 32 Grassmann-odd coordinates coupled to a corresponding NS-NS 2-form superfield depending on 32 θs. In this paper we present this action.

Likewise, a D2-brane which is embedded purely in an AdS_4 subspace[4] of $AdS_4 \times CP^3$ cannot be described by the D2-brane action based on the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ supercoset, since the embedding is incompatible with the kappa-symmetry gauge fixing[5] of the corresponding Green-Schwarz-type D2-brane action [15–17]. Other examples of this situation are D2- and D4-branes partially moving in AdS_4 and wrapping the 2-cycle in CP^3 associated with the CP^3 Kähler form J. Thus, to describe a general D-brane configuration in $AdS_4 \times CP^3$ one needs once again an explicit form of its action in the $AdS_4 \times CP^3$ superspace with 32 Grassmann-odd coordinates coupled to the corresponding NS-NS and RR superfields depending on 32 θs.

The main result of this paper is the explicit construction of the complete $AdS_4 \times CP^3$ superspace including all of the 32 Grassmann-odd coordinates. Unlike for most of the supergeometries studied previously in the literature, this type IIA $AdS_4 \times CP^3$ superspace is not a coset superspace, but we can nevertheless completely characterize its supergeometry. Having determined the supervielbeins of this superspace and the corresponding NS-NS and RR gauge superfields, we explicitly write down the general Green-Schwarz-type actions for

[4] An example of this situation is the D2-brane with $AdS_2 \times S^1 \subset AdS_4$ worldvolume [8], which corresponds to a disorder loop operator in the ABJM theory, and another example is the D2-brane at the Minkowski boundary of AdS_4.

[5] The discussion of the problem of fixing κ-symmetry in the D0– and D2-brane actions in $AdS_4 \times CP^3$ superspaces has been done in collaboration with P. Fré and P.A. Grassi.

the type IIA superstring and D-branes in $AdS_4 \times CP^3$. We analyze the classical equations of motion of the superstring in different submanifolds of the $AdS_4 \times CP^3$ superspace. On the submanifold described by the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ coset superspace, the classical superstring equations of motion are integrable [27, 28], generalizing the corresponding result found by Bena, Polchinski and Roiban for the type IIB superstring propagating on the $AdS_5 \times S^5$ supercoset [32]. However, we find that there is a submanifold in the $AdS_4 \times CP^3$ superspace that is described by a "twisted" $\mathrm{OSp}(2|4)/\mathrm{SO}(2) \times \mathrm{SO}(1,3)$ superspace, which is not a supercoset, and the ingredients used to prove integrability found in [32] do not directly apply to this sector of the theory. Therefore, it remains an important open problem to determine whether the complete set of classical equations of motion of the Green-Schwarz superstring propagating on the $AdS_4 \times CP^3$ superspace is classically integrable. The fact that the $AdS_4 \times CP^3$ superspace with 32 fermionic directions is not a supercoset requires more general techniques to prove classical integrability.

The explicit form of the supervielbeins and superconnections describing the $AdS_4 \times CP^3$ superspace are obtained by performing the Kaluza-Klein reduction of the supergeometry of the supercoset $\mathrm{OSp}(8|4)/\mathrm{SO}(7) \times \mathrm{SO}(1,3)$, which is a solution of the D=11 superfield supergravity constraints corresponding to the maximally supersymmetric $AdS_4 \times S^7$ vacuum of eleven dimensional supergravity. It is well known since the first intensive studies of flux compactifications of D=10 and D=11 supergravities that type IIA supergravity vacua[6] can be lifted to corresponding bosonic solutions of D=11 supergravity by constructing U(1) fibrations over the ten dimensional manifold characterizing the type IIA supergravity solutions [39–41]. For example, the 7-sphere is a U(1) Hopf fibration over CP^3, and therefore the $AdS_4 \times CP^3$ solution of the bosonic equations of type IIA supergravity [36] is directly related to the Freund-Rubin $AdS_4 \times S^7$ solution of the bosonic $D = 11$ supergravity equations of motion by reducing along the U(1)-fiber direction of the S^7 [40, 41]. For recent generalizations of these old results to the description of new compactified type IIA vacua see e.g. [42–45].

Extending the Kaluza-Klein reduction to superspace is much more subtle. When the Hopf fibration of $AdS_4 \times S^7$ is lifted to $D = 11$ superspace, such that $AdS_4 \times S^7$ becomes the bosonic subspace of the $\mathrm{OSp}(8|4)/\mathrm{SO}(7) \times \mathrm{SO}(1,3)$ supercoset, the supervielbeins of the supercoset do not come in a form suitable for performing the dimensional reduction of the D=11 superspace down to the type IIA D=10 superspace (see [46] for the general prescription for performing such a superspace reduction and [47] for more details). As we

[6]Let us here make the historical remark that the compactified vacuum solutions of type IIA supergravity corresponding to a direct product of AdS_4 and a compact manifold M^6 [33–36] were obtained by a combination of two mechanisms of spontaneous (flux) compactification proposed in 1980. One of the mechanisms was due to Freund and Rubin [37] in which the compactification of a D-dimensional theory into an $AdS_n \times M^{D-n}$ manifold takes place as a result of the interaction of gravity with a closed n-form or $(D-n)$-form field strength of an antisymmetric gauge field. Another mechanism was proposed by Volkov and Tkach [38]. Volkov and Tkach showed that in an interacting theory of gravity with Yang-Mills fields the compactification of extra dimensions may take place into coset spaces when components of the Yang-Mills fields take the same values as some components of the spin connection of the compactified manifold. The field strengths of the vacuum configurations of the Yang-Mills fields are (using the modern terminology) topologically nontrivial fluxes supported by compact subspaces.

JHEP03(2009)015

shall show, to get the $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ supervielbeins in the Kaluza-Klein-like form one should perform a "twist" of their components along the AdS_4 and the U(1)-fiber directions, or in other words perform a local Lorentz rotation in the 5-dimensional subspace tangent to AdS_4 and the U(1)-fiber direction along S^7. We should stress that such a transformation is not part of the isometry of the $AdS_4 \times S^7$ solution and should be regarded as an appropriate choice of a different supervielbein basis of $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ which has the Kaluza-Klein form compatible with the Hopf fibration. Note that by orbifolding the $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ supercoset by $Z_k \subset \mathrm{U}(1)$, where U(1) is the commutant of SU(4) in SO(8), one gets the supergeometry corresponding to the superspace with an $AdS_4 \times S^7/Z_k$ bosonic subspace, a background of eleven dimensional supergravity which preserves 24 supersymmetries (for $k > 2$) and is the near horizon geometry of N M2-branes probing the C^4/Z_k singularity.

Having obtained the complete supergeometry with 32 fermionic directions describing the $AdS_4 \times CP^3$ solution of type IIA supergravity, one can then use it to write down the Green-Schwarz-type actions for the type IIA superstring and D-branes (or the pure spinor action for the superstring) depending on all 32 fermions. This gives the complete and consistent description of these objects in the type IIA $AdS_4 \times CP^3$ superbackground. The complete form of the Green-Schwarz action provides a systematic framework in which to study the $AdS_4/CFT3$ correspondence and other problems.

The plan of the rest of the paper is as follows. In section 2, for the reader's convenience, we summarize our results and write down the explicit supergeometry for the type IIA $AdS_4 \times CP^3$ background. The details of our computations appear in the rest of the paper. In section 3 we write down the actions for the superstring and D-branes in this superbackground. We also analyze the motion of the string in submanifolds of the $AdS_4 \times CP^3$ superspace and note that the string equations of motion in a certain subspace are integrable [27, 28]. We find, however, that there is a submanifold in superspace for which the criteria found to prove integrability in [32] are not satisfied. So whether the Green-Schwarz superstring in $AdS_4 \times CP^3$ is integrable remains to be proven. In section 4 we describe a coset space realization of S^7 as a U(1) bundle over CP^3. In section 5 we lift the Hopf fibration description of the S^7 to $D = 11$ superspace and show that the associated supervielbeins and superconnections can be brought to the Kaluza-Klein form by performing a particular local Lorentz transformation, which allows us to read off the supergeometry for the type IIA $AdS_4 \times CP^3$ background. The main notation, conventions and some computations are presented in the appendices A–C.

2 $AdS_4 \times CP^3$ superspace with 32 Grassmann-odd directions

In this section we summarize our main result, namely, the construction of the superspace which has 32 Grassmann-odd directions, contains $AdS_4 \times CP^3$ as its bosonic part and solves the type IIA supergravity constraints [15, 17, 47, 48]. The derivation of this result is given in sections 4–5.

The type IIA superspace of interest is parametrized by 10 bosonic coordinates $X^M = (x^m, y^{m'})$, where x^m $(m = 0, 1, 2, 3)$ and $y^{m'}$ $(m' = 1, \dots 6)$ parametrize AdS_4 and CP^3

respectively, and by 32-fermionic coordinates $\theta^{\underline{\mu}} = (\theta^{\mu\mu'})$, which combine into the supercoordinates $Z^{\mathcal{M}} = (x^m,\, y^{m'},\, \theta^{\mu\mu'})$. The spinor indices $\mu = 1, 2, 3, 4$, and $\mu' = 1, \ldots, 8$ label, respectively, an SO$(2,3)$ and SO(6) spinor representation.

The 32 fermionic coordinates $\theta^{\mu\mu'}$ split into 24 coordinates $\vartheta^{\mu m'}$, which correspond to the 24 unbroken supersymmetries of the $AdS_4 \times CP^3$ background, and 8 coordinates $\upsilon^{\mu i}$ $(i = 1, 2)$ corresponding to the 8 broken supersymmetries.[7]

The type IIA supervielbeins are[8]

$$\mathcal{E}^{\mathcal{A}} = dZ^{\mathcal{M}}\, \mathcal{E}_{\mathcal{M}}{}^{\mathcal{A}}(Z) = (\mathcal{E}^{A},\, \mathcal{E}^{\underline{\alpha}})\,, \tag{2.1}$$

where

$$\mathcal{E}^{A}(Z) = (\mathcal{E}^{a},\, \mathcal{E}^{a'}) \qquad a = 0, 1, 2, 3, \quad a' = 1, \ldots, 6 \tag{2.2}$$

are the vector supervielbeins in the tangent space of $AdS_4 \times CP^3$ and

$$\mathcal{E}^{\underline{\alpha}}(Z) = \mathcal{E}^{\alpha\alpha'} = (\mathcal{E}^{\alpha a'},\, \mathcal{E}^{\alpha i}) \qquad \alpha = 1, 2, 3, 4, \quad \alpha' = 1, \ldots, 8\,, \quad i = 1, 2 \tag{2.3}$$

are the fermionic supervielbeins which split into 24 along the unbroken supersymmetry directions and eight along the broken ones. (The spinor indices $\alpha = 1, 2, 3, 4$, and $\alpha' = 1, \ldots, 8$ label, respectively, an SO$(1,3)$ and a U(3) representation.) The supervielbeins (2.2) and (2.3) are expressed in terms of the supervielbeins $E^{A}(x, y, \vartheta)$, $E^{\alpha a'}(x, y, \vartheta)$ and the U(1) connection $A(x, y, \vartheta)$ of the OSp$(6|4)/$U$(3) \times$ SO$(1,3)$ supercoset, whose fermionic coordinates are $\vartheta^{\alpha a'}$, but the former also depend on the 8 additional fermionic coordinates $\upsilon^{\alpha i}$ as follows[9]

$$\begin{aligned}
\mathcal{E}^{a'}(x, y, \vartheta, \upsilon) &= e^{\frac{1}{3}\phi(\upsilon)} \left(E^{a'}(x, y, \vartheta) - 2\upsilon\, \frac{\sinh m}{m} \gamma^{a'} \gamma^5\, E(x, y, \vartheta) \right)\,, \\
\mathcal{E}^{a}(x, y, \vartheta, \upsilon) &= e^{\frac{1}{3}\phi(\upsilon)} \left(E^{b}(x, y, \vartheta) - 4\upsilon\gamma^{b}\, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\, D\upsilon \right) \Lambda_b{}^{a}(\upsilon) \\
&\quad - e^{-\frac{1}{3}\phi(\upsilon)} \left(A(x, y, \vartheta) - 4i\upsilon\, \varepsilon\gamma^5\, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\, D\upsilon \right) E_7{}^{a}(\upsilon)\,, \qquad\qquad (2.4) \\
\mathcal{E}^{\alpha i}(x, y, \vartheta, \upsilon) &= e^{\frac{1}{6}\phi(\upsilon)} \left(\frac{\sinh \mathcal{M}}{\mathcal{M}}\, D\upsilon \right)^{\beta j} S_{\beta j}{}^{\alpha i}(\upsilon) - i\, e^{\phi(\upsilon)} \mathcal{A}_1(x, y, \vartheta, \upsilon)\, (\gamma^5 \varepsilon \lambda(\upsilon))^{\alpha i}\,, \\
\mathcal{E}^{\alpha a'}(x, y, \vartheta, \upsilon) &= e^{\frac{1}{6}\phi(\upsilon)}\, E^{\gamma b'}(x, y, \vartheta) \left(\delta_{\gamma}{}^{\beta} - 8 \left(i\gamma^5\, \upsilon\, \frac{\sinh^2 m/2}{m^2} \right)_{\gamma i} \upsilon^{\beta i} \right) S_{\beta b'}{}^{\alpha a'}(\upsilon)\,,
\end{aligned}$$

where $E(x, y, \vartheta)$ in the second term of the first expression is the spinor one-form $E^{\gamma b'}(x, y, \vartheta)$ which also appears in the last expression of (2.4).

[7]This splitting is carried out by applying the projectors (5.1) and (5.4) on $\theta^{\underline{\mu}}$ (See appendices A and C for more details).

[8]Our convention for the essential torsion constraint of IIA supergravity is $T_{\underline{\alpha\beta}}{}^{A} = 2\Gamma^{A}_{\underline{\alpha\beta}}$. This choice is related to the form of the OSp$(8|4)$ algebra (appendix B, eq. (B.7)) and differs from that of [47] by the factor $2i$.

[9]These are the formulas for the case when k, corresponding to the order of the Z_k orbifold of the S^7 and the type IIA RR two-form flux through CP^3, is set to $k = 1$. The formulas for general k are obtained by making the following rescaling: $\Phi \to \frac{1}{k}\Phi$, $E_7{}^{a} \to \frac{1}{k} E_7{}^{a}$ and $e^{\frac{2}{3}\phi} \to \frac{1}{k} e^{\frac{2}{3}\phi}$.

The type IIA RR one-form gauge superfield is

$$\mathcal{A}_1(x,y,\vartheta,\upsilon) = e^{-\frac{4}{3}\phi(\upsilon)} \left[\left(A(x,y,\vartheta) - 4i\upsilon\,\varepsilon\gamma^5 \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} D\upsilon \right) \Phi(\upsilon) + \left(E^a(x,y,\vartheta) - 4\upsilon\gamma^a \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} D\upsilon \right) E_{7a}(\upsilon) \right] \tag{2.5}$$

with the field strength $F_2 = d\mathcal{A}_1$. The RR four-form and NS-NS three-form superfield strengths are given by

$$F_4 = d\mathcal{A}_3 - \mathcal{A}_1\, H_3 = -\frac{1}{4!}\mathcal{E}^d\mathcal{E}^c\mathcal{E}^b\mathcal{E}^a \left(6e^{-2\phi}\Phi\varepsilon_{abcd}\right) + \frac{1}{2}\mathcal{E}^B\mathcal{E}^A\mathcal{E}^{\underline{\beta}}\mathcal{E}^{\underline{\alpha}}e^{-\phi}(\Gamma_{AB})_{\underline{\alpha\beta}} \tag{2.6}$$
$$H_3 = dB_2 = -\frac{1}{3!}\mathcal{E}^c\mathcal{E}^b\mathcal{E}^a \left(6e^{-\phi}\varepsilon_{abcd}E_7{}^d\right) + \mathcal{E}^A\mathcal{E}^{\underline{\beta}}\mathcal{E}^{\underline{\alpha}}(\Gamma_A\Gamma_{11})_{\underline{\alpha\beta}} - \mathcal{E}^B\mathcal{E}^A\mathcal{E}^{\underline{\alpha}}(\Gamma_{AB}\Gamma^{11}\lambda)_{\underline{\alpha}}\,,$$

where Γ_A and Γ_{11} are 32×32 gamma-matrices which in the $AdS_4\times CP^3$ background are convenient to represent as a direct product of 4×4 and 8×8 gamma-matrices (see eq. (A.8) of appendix A).

The gauge potentials of (2.6), which appear in the superstring and D-brane actions, can be computed by a standard procedure as follows:

$$B_2 = b_2 + \int_0^1 dt\, i_\theta H_3(x,y,t\theta)\,, \qquad \theta = (\vartheta,\upsilon) \tag{2.7}$$

$$\mathcal{A}_3 = a_3 + \int_0^1 dt\, i_\theta \left(F_4 + \mathcal{A}_1 H_3\right)(x,y,t\theta)\,, \tag{2.8}$$

where b_2 and a_3 are the purely bosonic parts of the gauge potentials and i_θ means the inner product with respect to $\theta^{\mu\mu'}$. Note that b_2 is pure gauge in the $AdS_4\times CP^3$ solution.[10]

In eqs. (2.4)–(2.5)

$$D\upsilon = \left(d + iE^a(x,y,\vartheta)\gamma^5\gamma_a - \frac{1}{4}\Omega^{ab}(x,y,\vartheta)\gamma_{ab}\right)\upsilon\,, \tag{2.9}$$

where $E^{a'}(x,y,\vartheta)$, $E^a(x,y,\vartheta)$ and $\Omega^{ab}(x,y,\vartheta)$ are, respectively, the CP^3 and AdS_4 part of the supervielbein and the SO(1,3) connection of the OSp(6|4)/U(3) × SO(1,3) supercoset, while $E^{\alpha a'}(x,y,\vartheta)$ is its spinorial supervielbein. $A(x,y,\vartheta)$ is the U(1) connection on the OSp(6|4)/U(3) × SO(1,3) supercoset, which corresponds to the RR one-form gauge potential for this type IIA supergravity solution, while in the complete superspace it is given by (2.5). All these quantities are known explicitly and can be taken in any suitable form, which one can find, e.g. in [22, 27–29, 31] or in our appendix A, eq. (A.10). An appropriate choice of the supercoset representatives may drastically simplify their fermionic dependence (see e.g. [26]).

[10] To derive eqs. (2.7) and (2.8) one should use the fact that the coordinate variation of a differential superform $A(Z) = A(X,\theta)$ is $\delta A = i_{\delta Z}dA + d(i_{\delta Z}A)$. Then, rescaling $\theta \to t\theta$ in $A(X,\theta)$ and taking the derivative with respect to t, we have $\frac{d}{dt}A(X,t\theta) = i_\theta dA + d(i_\theta A)$, which upon integration over t gives eqs. (2.7) and (2.8), up to pure gauge terms.

The quantities $\Lambda_a{}^b(\upsilon)$ and $S_{\alpha\alpha'}{}^{\beta\beta'}(\upsilon)$ appearing in the above equations have the form

$$\begin{aligned}
\Lambda_a{}^b &= \delta_a{}^b - \frac{e^{-\frac{2}{3}\phi}}{e^{\frac{2}{3}\phi}+\Phi} E_{7a}\, E_7{}^b \\
S &= \frac{e^{-\frac{1}{3}\phi}}{\sqrt{2}}\left(\sqrt{e^{\frac{2}{3}\phi}+\Phi} - \frac{E_7{}^a\,\Gamma_a\Gamma_{11}}{\sqrt{e^{\frac{2}{3}\phi}+\Phi}}\right).
\end{aligned} \tag{2.10}$$

They generate the Lorentz transformation in the OSp(8|4)/SO(7)×SO(1,3) supergeometry which brings the $D=11$ superspace into the Kaluza-Klein form required to perform its dimensional reduction to the $D=10$ superspace (see section 5).

The function $\phi(\upsilon)$ is the dilaton superfield of the full type IIA superspace solution under consideration. The dilaton superfield depends only on the 8 Grassmann coordinates $\upsilon^{\alpha i}$ and has the following expression in terms of $E_7{}^a(\upsilon)$ and $\Phi(\upsilon)$

$$e^{\frac{2}{3}\phi(\upsilon)} = \sqrt{\Phi^2 + E_7{}^a\, E_7{}^b\, \eta_{ab}}\,. \tag{2.11}$$

The fermionic field $\lambda^{\alpha i}(\upsilon)$ describes the non-zero components of the dilatino superfield, which is defined by the equation [47]

$$\lambda_{\alpha i} = -\frac{1}{3} D_{\alpha i}\, \phi(\upsilon). \tag{2.12}$$

Other quantities appearing in eqs. (2.4)–(2.12), namely $\mathcal{M}$, m, $\Phi(\upsilon)$ and $E_7{}^a(\upsilon)$, whose geometrical and group-theoretical meaning is explained in section 5, are explicitly given in eqs. (5.30), (5.31).

One can notice that a distinctive feature of the $AdS_4\times CP^3$ IIA superspace with 32 Grassmann-odd directions compared to the coset superspace OSp(6|4)/U(3) × SO(1,3) with only 24 Grassmann-odd directions is that in the full superspace solution the dilaton, dilatino and the NS-NS 3-form superfield have non-zero values, and depend on the 8 fermionic coordinates which correspond to broken supersymmetries of the $AdS_4\times CP^3$ IIA supergravity solution.

For brevity we do not present here the explicit form of the superconnections of the $AdS_4\times CP^3$ superspace, since they are not required for the construction of the superstring and brane actions. When necessary, they can be directly recovered from the Cartan forms of OSp(8|4)/SO(7) × SO(1,3), as explained in section 5.

3 Actions for the type IIA superstring and D-branes in the complete $AdS_4\times CP^3$ superspace

3.1 Type IIA Green-Schwarz superstring

The Green-Schwarz superstring action [12] in a generic supergravity background is well-known [13] and its Nambu-Goto form is

$$S = -T\int d^2\xi\sqrt{-\det\left(\mathcal{E}_i{}^A\,\mathcal{E}_j{}^B\,\eta_{AB}\right)} + T\int B_2(\xi), \tag{3.1}$$

where T is the string tension, ξ^i $(i=0,1)$[11] are the string worldsheet coordinates, $\mathcal{E}_i{}^A = \partial_i Z^{\mathcal{M}}(\xi)\,\mathcal{E}_{\mathcal{M}}{}^A$ is the worldsheet pullback of the vector supervielbeins of type IIA supergravity and $B_2(\xi) = \frac{1}{2} d\xi^i\, d\xi^j\, \partial_i Z^{\mathcal{N}} \partial_j Z^{\mathcal{M}}\, B_{\mathcal{M}\mathcal{N}}(Z)$ is the worldsheet pullback of the NS-NS two-form superfield.

Provided that the superbackground satisfies the IIA supergravity constraints, the action (3.1) is invariant under kappa-symmetry transformations of the superstring coordinates $Z^{\mathcal{M}}(\xi)$ which for all known superbranes have the following generic form

$$\delta_\kappa Z^{\mathcal{M}}\,\mathcal{E}_{\mathcal{M}}{}^{\underline{\alpha}} = \frac{1}{2}(1+\bar{\Gamma})^{\underline{\alpha}}{}_{\underline{\beta}}\,\kappa^{\underline{\beta}}(\xi), \qquad \underline{\alpha} = 1,\ldots,32 \tag{3.2}$$

$$\delta_\kappa Z^{\mathcal{M}}\,\mathcal{E}_{\mathcal{M}}{}^{A} = 0, \qquad A = 0,1,\ldots,9 \tag{3.3}$$

where $\kappa^{\underline{\alpha}}(\xi)$ is a 32-component spinor parameter and $\frac{1}{2}(1+\bar{\Gamma})^{\underline{\alpha}}{}_{\underline{\beta}}$ is a spinor projection matrix (such that $\bar{\Gamma}^2 = 1$) specific to each type of superbrane.

In the case of the type IIA superstring the matrix $\bar{\Gamma}$ is

$$\bar{\Gamma} = \frac{1}{2\sqrt{-\det g_{ij}}}\,\epsilon^{ij}\,\mathcal{E}_i{}^A\,\mathcal{E}_j{}^B\,\Gamma_{AB}\,\Gamma_{11}, \tag{3.4}$$

where

$$g_{ij}(\xi) = \mathcal{E}_i{}^A\,\mathcal{E}_j{}^B\,\eta_{AB} \tag{3.5}$$

is the induced metric on the worldsheet of the string.

To describe the type IIA superstring in the complete $AdS_4 \times CP^3$ superspace we should just substitute into the above equations the explicit form of the vector supervielbeins $\mathcal{E}^A$ and the NS-NS two-form B_2 given in eqs. (2.4) and (2.7).

Note that kappa-symmetry allows one to eliminate 16 out of 32 fermionic degrees of freedom of the superstring. It can be used to simplify and reduce the form of the supervielbein pullbacks and, as a consequence, the form of the string action. For example, one might be willing, by using kappa-symmetry, to get rid of the 8 fermionic coordinates $\upsilon^{\alpha i}$ corresponding to the 8 broken supersymmetries of $AdS_4 \times CP^3$. As a result of such a partial gauge fixing, one arrives at the superstring action of [27, 28, 31], which can be described by a sigma-model on the OSp(6|4)/U(3) × SO(1,3) supercoset. However, the kappa-symmetry gauge fixing which completely eliminates $\upsilon^{\alpha i}$ is only possible when the kappa-symmetry projector (3.2), (3.4) does *not* commute with the projector $\mathcal{P}_2$, eq. (5.4), which singles out 8 out of 32 fermionic coordinates. This is not the case, for example, when the string moves entirely in the AdS_4 space. In this case $[\bar{\Gamma}, \mathcal{P}_2] = 0$, and kappa-symmetry can only eliminate half of the eight $\upsilon^{\alpha i}$'s. Hence, such configurations of the string in $AdS_4 \times CP^3$ cannot be described by the sigma-model action based on the OSp(6|4)/U(3) × SO(1,3) supercoset and one should use the action (3.1) in the full superspace.

[11]Since we have exhausted a finite number of letters which are at our disposal to define different types of indices, here we use the letters i, j to denote the worldvolume indices. We believe that this will not cause confusion with the same letters used in the previous section to define SO(2) ⊂ SO(8) indices.

JHEP03(2009)015

3.1.1 Classical integrability of Green-Schwarz action in $AdS_4 \times CP^3$ superspace

f The explicit form of the Green-Schwarz action in $AdS_4 \times CP^3$ allows for the study of the most general solution to the string equations of motion in this background. Furthermore, having the complete action in superspace provides us with a systematic framework in which to compute quantum corrections to any classical string solution. Classical string solutions together with their quantum corrections play an important role in the AdS/CFT correspondence, as they correspond to certain "long" operators in the gauge theory (see e.g. [49, 50]).

In [32] it has been shown that the Green-Schwarz action in $AdS_5 \times S^5$ [21] is classically integrable.[12] This can be proven by explicitly constructing a Lax connection representation of the superstring equations of motion, such that flatness of the Lax connection $\mathcal{L}_i$

$$\partial_i \mathcal{L}_j - \partial_j \mathcal{L}_i - [\mathcal{L}_i, \mathcal{L}_j] = 0 \tag{3.6}$$

implies the superstring equations of motion. The crucial ingredients in the construction of the Lax connection are the Cartan forms in the $AdS_5 \times S^5$ coset superspace $\mathrm{SU}(2,2|4)/\mathrm{SO}(1,4) \times \mathrm{SO}(5)$ and the existence of a Z_4 automorphism of the $\mathrm{SU}(2,2|4)$ algebra. One can then construct the conserved charges of the integrable model from the holonomy of the Lax connection (see [32] for more details).

The construction in [32] guarantees that any sigma-model based on a supercoset G/H is classically integrable as long as the superalgebra G admits a Z_4 grading. This general construction provides a simple diagnostic for determining whether a large class of supercoset models are classically integrable. In this paper, however, we have shown that the complete $AdS_4 \times CP^3$ Type IIA superspace is not given by a coset superspace. This implies that the technique introduced in [32] does not directly apply, as we cannot longer construct a candidate Lax connection $\mathcal{L}_i$ from the Cartan forms of the supercoset. Nevertheless, we can study the various allowed motions of the superstring along submanifolds of the complete $AdS_4 \times CP^3$ Type IIA superspace and analyze whether the equations of motion governing the allowed motions are classically integrable.

Wherever it is allowed, by partially fixing kappa-symmetry, we can set to zero the 8 fermionic coordinates $v^{\alpha i}$ which correspond to the 8 supersymmetries broken by the $AdS_4 \times CP^3$ background. This choice selects the submanifold $\mathcal{M}_{10,24}$ in the complete $AdS_4 \times CP^3$ superspace. In this submanifold, the superstring can move in the full $AdS_4 \times CP^3$ bosonic subspace (the string must propagate, however, both in AdS_4 and in CP^3 in order to be compatible with the gauge fixing [27]) but the motion of the string is restricted to a 24 dimensional fermionic submanifold of the superspace spanned by the coordinates $\vartheta^{\alpha a'}$. For these classical configurations, the complete $AdS_4 \times CP^3$ superspace found in this paper reduces to the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ coset superspace already considered in [27–29, 31]. For this family of classical solutions the Green-Schwarz action can be completely written down in terms of the Cartan forms of the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ supercoset, very much like for the type IIB superstring action on the $AdS_5 \times S^5$ coset superspace. Moreover, since the $\mathrm{OSp}(6|4)$ algebra admits a Z_4 automorphism, the construction in [32] can be carried

[12]See [51] for earlier work considering the classical integrability of the bosonic sigma-model.

over to this case to show that the classical equations of motion of the superstring in the subspace $\mathcal{M}_{10,24}$ of the complete $AdS_4 \times CP^3$ superspace is integrable [27, 28].[13]

The gauge fixed action with $\upsilon^{\alpha i} = 0$ is, however, incompatible with motions of the string e.g. purely in the AdS_4 geometry, which constitute an important family of classical solutions (see e.g. [49]). One can study these motions of the string by considering a submanifold $\mathcal{M}_{4,8}$ of the complete $AdS_4 \times CP^3$ superspace. This submanifold is spanned by the AdS_4 bosonic geometry and by the 8 dimensional fermionic space parametrized by the coordinates $\upsilon^{\alpha i}$.[14] It follows from our expressions for the $AdS_4 \times CP^3$ superspace, that the submanifold $\mathcal{M}_{4,8}$ can be associated with a "twisted coset" superspace OSp(2|4)/SO(2) $\times$ SO(1,3), where the Cartan forms are rotated by a local Lorentz transformation in $D = 11$ superspace, which was required to perform the Kaluza-Klein reduction to the $AdS_4 \times CP^3$ superspace. The twisting reflects the fact that the fermionic coordinates of this superspace correspond to 8 broken supersymmetries. Thus this superspace does not have superisometries. The supervielbeins and the Abelian one-form superfield in this "twisted coset" superspace have the following form (see eqs. (2.4) and (2.5))

$$\begin{aligned}
\mathcal{E}^a(x,\upsilon) &= e^{\frac{1}{3}\phi(\upsilon)} \left(e^b(x) - 4\upsilon\gamma^b \, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \, D\upsilon \right) \Lambda_b{}^a(\upsilon) \\
&\qquad + 4i\, e^{-\frac{1}{3}\phi(\upsilon)} \, \upsilon\, \varepsilon\gamma^5 \, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \, D\upsilon \, E_7{}^a(\upsilon)\,, \\
\mathcal{E}^{\alpha i}(x,\upsilon) &= e^{\frac{1}{6}\phi(\upsilon)} \left(\frac{\sinh \mathcal{M}}{\mathcal{M}} \, D\upsilon \right)^{\beta j} S_{\beta j}{}^{\alpha i}(\upsilon) - e^{\phi(\upsilon)} \mathcal{A}_1(x,\upsilon)\, (i\gamma^5 \varepsilon\lambda(\upsilon))^{\alpha i}\,, \\
\mathcal{A}_1(x,\upsilon) &= e^{-\frac{4}{3}\phi(\upsilon)} \left[\left(e^a(x) - 4\upsilon\gamma^a \, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \, D\upsilon \right) E_{7a} - 4i\upsilon\, \varepsilon\gamma^5 \, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \, D\upsilon \, \Phi(\upsilon) \right],
\end{aligned} \tag{3.7}$$

where

$$D\upsilon = \left(d + i e^a(x) \gamma^5 \gamma_a - \frac{1}{4} \omega^{ab}(x) \gamma_{ab} \right) \upsilon\,, \tag{3.8}$$

and $e^a(x)$ and $\omega^{ab}(x)$ are the AdS_4 vielbeins and connection respectively. The RR and NS-NS superfields in this four-dimensional supermanifold have the same form as in (2.6) but with the D=10 supervielbeins replaced with eqs. (3.7).

For comparison, let us present the supervielbeins for the conventional supercoset OSp(2|4)/SO(2) $\times$ SO(1,3)

$$\begin{aligned}
\mathcal{E}^a(x,\upsilon) &= e^a(x) - 4\upsilon\gamma^a \, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \, D\upsilon\,, \\
\mathcal{E}^{\alpha i}(x,\upsilon) &= \left(\frac{\sinh \mathcal{M}}{\mathcal{M}} \, D\upsilon \right)^{\alpha i}, \\
\mathcal{A}_1(x,\upsilon) &= -4i\upsilon\, \varepsilon\gamma^5 \, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} \, D\upsilon\,.
\end{aligned} \tag{3.9}$$

[13] In [52] the algebraic curve characterizing the classical solutions on this supercoset has been proposed.

[14] In this subspace the string worldsheet scalars $y^{m'}$ are constant and $\vartheta^{\alpha a'}$ are covariantly constant (Killing) spinors, $D\vartheta = 0$ (on the worldsheet).

JHEP03(2009)015

Since the "twisted" OSp(2|4)/SO(2) × SO(1, 3)" supermanifold is not a coset superspace, the criteria used in [32] to prove integrability of the classical equations of motion do not directly apply to this superspace. Therefore, it remains an important open problem to determine whether our explicit form of the Green-Schwarz action when restricted to $\mathcal{M}_{4,8}$ is classically integrable. The explicit expressions for the $AdS_4 \times CP^3$ supergeometry found in this paper provides a framework in which this question can be investigated.

Understanding the classical and quantum integrability of the superstring equations of motion in the $AdS_4 \times CP^3$ superspace also provides an important clue in determining whether the planar dilatation operator of the holographic dual ABJM $\mathcal{N} = 6$ Chern-Simons theory is integrable to all orders in the 't Hooft coupling. Integrability of the two-loop ABJM dilatation operator has been exhibited in [54, 55, 57] and a conjecture for the all loop Bethe ansatz has been made in [58]. However, unlike for the maximally supersymmetric AdS_5/CFT_4 duality, the magnon dispersion relation acquires non-trivial quantum corrections both at strong coupling as well as in the weak coupling regime [9, 10, 55, 56, 58, 59], significantly complicating the AdS_4/CFT_3 analysis with respect to the case of the AdS_5/CFT_4 duality. More work is needed to convincingly argue that the ABJM planar dilatation operator is exactly integrable. Determining whether the ABJM theory is exactly integrable in the planar limit and whether the Green-Schwarz superstring in the $AdS_4 \times CP^3$ superspace is integrable remain two important problems to resolve in this new holographic correspondence.

3.2 Type IIA D-branes

The action for a Dp-brane ($p = 0, 2, 4, 6, 8$) in a general type IIA supergravity background [15–17] in the string frame has the form

$$S = -T_p \int d^{p+1}\xi \, e^{-\phi} \sqrt{-\det(g_{ij} + \mathcal{F}_{ij})} + T_p \int e^{\mathcal{F}_2} \wedge \mathbb{A}|_{p+1} \, , \tag{3.10}$$

where T_p is the tension of the Dp-brane,

$$g_{ij}(\xi) = \mathcal{E}_i{}^A \, \mathcal{E}_j{}^B \, \eta_{AB} \qquad i, j = 0, \ldots, p \tag{3.11}$$

is the induced metric on the Dp-brane worldvolume and

$$\mathcal{F}_2 = d\mathcal{V} - B_2 \tag{3.12}$$

is the field strength of the worldvolume Born-Infeld gauge field $\mathcal{V}_i(\xi)$ extended by the pullback of the NS-NS two-form. In the second term of eq. (3.10), the Wess-Zumino term, $|_{p+1}$ means that we must pick only the terms which are $(p + 1)$-forms in the D-brane worldvolume from the *formal* sum of the forms of different degrees

$$\begin{aligned} e^{\mathcal{F}_2} &= 1 + \mathcal{F}_2 + \frac{1}{2}\mathcal{F}_2\mathcal{F}_2 + \frac{1}{3!}\mathcal{F}_2\mathcal{F}_2\mathcal{F}_2 + \frac{1}{4!}\mathcal{F}_2\mathcal{F}_2\mathcal{F}_2\mathcal{F}_2 + \frac{1}{5!}\mathcal{F}_2\mathcal{F}_2\mathcal{F}_2\mathcal{F}_2\mathcal{F}_2 = \sum_{k=0}^{5} \frac{1}{n!}(\mathcal{F}_2)^n \, , \\ \mathbb{A} &= \mathcal{A}_1 + \mathcal{A}_3 + \mathcal{A}_5 + \mathcal{A}_7 + \mathcal{A}_9 = \sum_{n=0}^{4} \mathcal{A}_{2n+1} \, , \end{aligned} \tag{3.13}$$

where $\mathcal{A}_n$ are the type IIA supergravity RR superforms and their Hodge duals.

The action (3.10) is invariant under the kappa-symmetry transformations (3.2)–(3.3) provided that the superbackground satisfies the type IIA supergravity constraints and the Born-Infeld field transforms as follows

$$\delta_\kappa \mathcal{V} = i_{\delta_\kappa} B_2 \qquad \Rightarrow \qquad \delta_\kappa \mathcal{F}_2 = -i_{\delta_\kappa} H_3 \,. \tag{3.14}$$

The explicit form of the kappa-symmetry projection matrix Γ is given in [15–17].

To describe the Dp-branes in the $AdS_4 \times CP^3$ superbackground one should substitute into the above expressions the explicit form of the supervielbeins, NS-NS and RR forms given in (2.4), (2.7), (2.5) and (2.8). As in the superstring case, one can verify that for the D0-brane and a D2-brane moving entirely in the AdS_4 space, the corresponding kappa-symmetry projector commutes with the projector $\mathcal{P}_2$ (5.4) which singles out the 8 fermionic coordinates $\upsilon^{\alpha i}$ in superspace. For these configurations, kappa-symmetry cannot eliminate all eight $\upsilon^{\alpha i}$'s, but only half of them, just like for the case of the superstring moving entirely in AdS_4. Therefore, such configurations of D0 and D2-branes cannot be described by sigma-models based on the supercoset OSp(6|4)/U(3) $\times$ SO(1,3), and one should use the complete IIA superspace constructed in this paper.[5] In particular, one should use this complete superspace for studying the AdS_4/CFT_3 correspondence for the D2-branes placed at the boundary of AdS_4 as well as for the D2-branes corresponding to vortex loop operators in the boundary field theory [8].

Other examples of brane configurations for which kappa-symmetry cannot completely remove the 8 'broken' fermionic coordinates are D2– and a D4-branes wrapping the 2-cycle of CP^3 associated with the CP^3 Kähler form J and moving in AdS_4.

In the next sections we shall explain in detail the construction of the complete type IIA $AdS_4 \times CP^3$ superspace which we summarized in section 2.

4 Coset space realization of S^7 as a fibration over CP^3

We construct the complete $D = 10$ $AdS_4 \times CP^3$ superspace by dimensional reduction of the $D = 11$ supercoset OSp(8|4)/SO(7) $\times$ SO(1,3) whose supervielbeins and superconnection have a fiber bundle form, generalizing to superspace the Hopf fibration form of the metric and connection of the 7-sphere. So let us start by reviewing the Hopf fiber bundle structure of the 7-sphere by considering it as a coset space.

S^7 can be realized as the symmetric space SO(8)/SO(7), however this realization does not provide us directly with the desired Hopf fibration form of its vielbein and connection. The coset realization of S^7 exhibiting its structure as a Hopf fibration over CP^3 is the coset space $\frac{\mathrm{SU}(4)\times \mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$. Note that this is not a symmetric space.[15] On the other hand, CP^3 is a symmetric space realized as the coset $\frac{\mathrm{SU}(4)}{\mathrm{SU}(3)\times \mathrm{U}(1)}$. The isometry group SU(4) $\times$ U(1) $\simeq$ SO(6) $\times$ SO(2) of the coset $\frac{\mathrm{SU}(4)\times \mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$ should be considered as a subgroup of SO(8), SU(3) is a subgroup of SU(4) and $U'(1)$, in the denominator, is generated as follows. Let T_2 be the generator of U(1) $\simeq$ SO(2) in the numerator of the coset $\frac{\mathrm{SU}(4)\times \mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$ and let T_1 be the

[15] A nice concise review of geometry of coset spaces the reader may find e.g. in [60].

U(1) subgroup of SU(4) which commutes with SU(3). Then the stability subgroup $U'(1)$ is generated by

$$T' = \frac{3}{4}(T_2 - T_1) \tag{4.1}$$

and the generator

$$P_7 = \frac{1}{4}(3T_1 + T_2) \tag{4.2}$$

corresponds to the 7th (U(1)-fiber) direction of S^7. The inverse expressions are

$$T_1 = P_7 - \frac{1}{3}T', \qquad T_2 = P_7 + T'\,. \tag{4.3}$$

In terms of generators of the SO(8) algebra (See appendices), the above generators are

$$T' = -\frac{1}{2}\,J^{a'b'}\,M_{a'b'}, \qquad P_7 = -M_{78} \tag{4.4}$$

where $M_{a'b'}$ are the SO(6) generators and $J_{a'b'}$ are the components of the Kähler form on CP^3 satisfying the relations

$$J_{a'b'} = -J_{b'a'}\,, \qquad J_{a'c'}\,J^{c'}{}_{b'} = -\delta_{a'b'}\,, \qquad \epsilon_{a'b'c'd'e'f'}\,J^{a'b'}\,J^{c'd'} = 8\,J_{e'f'}. \tag{4.5}$$

To get the 'fiber bundle form' of the vielbein and connection of the 7-sphere we choose the following coset representative of $\frac{\mathrm{SU}(4)\times\mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$

$$K = e^{\,y^{m'}P_{m'}}\,e^{\,zT_2} = e^{\,y^{m'}P_{m'}}\,e^{\,zP_7}\,e^{\,zT'}, \tag{4.6}$$

where $P_{m'}$ are the generators corresponding to the coset $CP^3 = \frac{\mathrm{SU}(4)}{\mathrm{SU}(3)\times\mathrm{U}(1)}$ parametrized by coordinates $y^{n'}$ $(n' = 1,\ldots,6)$ and z is the U(1) fiber coordinate of S^7 (associated with the generator P_7) so that $(y^{n'},\, z)$ are the seven local coordinates on the S^7. Note that $e^{\,zT'}$ in (4.6) plays the role of a compensating local transformation of the stability subgroup $U'(1)$.

The commutators of $P_{a'}$ close on the SU(3) generators L_I $(I = 1,\ldots 8)$ and the U(1) generator T_1. Altogether $P_{a'}$, L_I and T_1 form the SU(4) algebra

$$[P_{a'}, P_{b'}] = C_{a'b'}{}^{I}L_I + 2J_{a'b'}\,T_1, \quad [P_{a'}, L_I] = C_{a'I}{}^{b'}\,P_{b'}, \quad [P_{a'}, T_1] = -\frac{4}{3}\,J_{a'b'}\,P^{b'}\,, \tag{4.7}$$

$$[L_I, L_J] = C_{IJ}{}^{K}L_K\,, \qquad [L_I, T_1] = 0\,, \tag{4.8}$$

where $C_{Ia'b'}$, C_{IJK} and $2\,J_{a'b'}$ are the structure constants of the SU(4) algebra. In terms of SO(8) generators, T_1 was given in (4.3)–(4.4), and $P_{a'}$ and $C_{a'b'}{}^{I}L_I$ are (See appendix C for more details)

$$P_{a'} = -M_{a'8} + J_{a'}{}^{b'}\,M_{b'7}\,, \tag{4.9}$$

$$C_{a'b'}{}^{I}L_I = (\delta^{c'}_{a'}\,\delta^{d'}_{b'} + J_{a'}{}^{c'}\,J_{b'}{}^{d'})M_{c'd'} - \frac{1}{3}\,J_{a'b'}\,J^{c'd'}\,M_{c'd'}. \tag{4.10}$$

The Cartan form $K^{-1}dK$ determines the vielbeins and the $\mathrm{SU}(3)\times U'(1)$ connections on $\frac{\mathrm{SU}(4)\times\mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$

$$
\begin{aligned}
K^{-1}dK \,=\, & dy^{n'} e_{n'}{}^{a'}(y)\, P_{a'} + (dz + dy^{n'} A_{n'}(y))\, P_7 \\
& + dy^{n'} \omega_{n'}{}^{I}(y) L_I + \left(dz - \frac{1}{3}\, dy^{n'} A_{n'}(y)\right) T'\,,
\end{aligned} \tag{4.11}
$$

where

$$
e^{\hat{a}'} = \left(e^{a'},\, e^7\right), \qquad e^{a'} = dy^{n'}\, e_{n'}{}^{a'}(y)\,, \qquad e^7 = dz + dy^{n'}\, A_{n'}(y) \tag{4.12}
$$

are the S^7 vielbeins, with $e^{a'}(y)$ and $A(y)$ being associated with the vielbein and U(1) connection on CP^3, and

$$
\omega^I = dy^{n'}\, \omega_{n'}{}^{I}(y)\,, \qquad \omega' = dz - \frac{1}{3}\, dy^{n'} A_{n'}(y) \tag{4.13}
$$

are the SU(3) and $U'(1)$ connections respectively.

With the connections defined as in eq. (4.13), the coset space $\frac{\mathrm{SU}(4)\times\mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$ has torsion. This is because its stability subgroup $U'(1)$ is associated with the generator T' defined in eq. (4.1). One can see this analyzing the Maurer-Cartan equation

$$
d(K^{-1}dK) - (K^{-1}dK)\,(K^{-1}dK) = 0 \tag{4.14}
$$

from which follows, in particular, that

$$
D\, e^{a'} \equiv de^{a'} + e^{b'}\, \omega^I\, C_{Ib'}{}^{a'} - e^{b'}\, J_{b'}{}^{a'}\, \omega' = -\, e^{b'}\, e^7\, J_{b'}{}^{a'} \;= \frac{1}{2}\, e^{\hat{b}'} e^{\hat{c}'}\, T_{\hat{c}'\hat{b}'}{}^{a'}\,, \tag{4.15}
$$

$$
d\, e^7 = e^{b'} e^{c'}\, J_{b'c'} = \frac{1}{2}\, e^{\hat{b}'}\, e^{\hat{c}'}\, T_{\hat{c}'\hat{b}'}{}^{7}\,, \tag{4.16}
$$

where $T_{\hat{b}'\hat{c}'}{}^{\hat{a}'}$ ($\hat{a}' = (a', 7)$ etc.) are the components of the torsion tensor of the coset manifold $\frac{\mathrm{SU}(4)\times\mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$. To make the geometry on this manifold torsion-free, as in the standard Riemannian case, we should redefine its connection as follows

$$
\Omega^{\hat{a}'\hat{b}'} = (\Omega^{a'b'},\, \Omega^{a'7})\,, \tag{4.17}
$$

where

$$
\Omega^{a'b'} = \omega^I C_I{}^{a'b'} - \omega'\, J^{a'b'} = \omega^{a'b'} - e^7\, J^{a'b'}\,, \qquad \Omega^{a'7} = -\Omega^{7a'} = e^{b'}\, J_{b'}{}^{a'} \tag{4.18}
$$

while

$$
\omega^{a'b'} = \omega^I C_I{}^{a'b'} + \frac{4}{3}\, dx^{n'} A_{n'} J^{a'b'} \tag{4.19}
$$

is the Riemannian U(3) connection on CP^3.

One can show that the curvature of the $\frac{\mathrm{SU}(4)\times\mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$ coset associated with the connection (4.17) is

$$
d\Omega^{\hat{a}'\hat{b}'} + \Omega^{\hat{a}'}{}_{\hat{c}'}\, \Omega^{\hat{c}'\hat{b}'} = e^{\hat{a}'} e^{\hat{b}'}, \tag{4.20}
$$

JHEP03(2009)015

where the vielbeins $e^{\hat{a}'}$ were defined in (4.12). We see that the curvature (4.20) is that of the round S^7 sphere.[16] This completes the demonstration that the Hopf fibration over CP^3 associated with the coset space $\frac{\mathrm{SU}(4)\times \mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)}$ and endowed with the Riemann connection and curvature is the 7-sphere having SO(8) isometry, which is enhanced with respect to the initial $\mathrm{SU}(4)\times\mathrm{U}(1)$ manifest symmetry of the coset.

The U(1) bundle realization (4.12) of the vielbeins of S^7 is very convenient for performing the Kaluza-Klein dimensional reduction of the $AdS_4\times S^7$ solution of $D=11$ supergravity down to the corresponding $AdS_4\times CP^3$ solution of IIA $D=10$ supergravity [40, 41]

$$D=11: \quad e^{\hat{A}}=(e^a,e^{\hat{a}'}) \qquad \Rightarrow \qquad D=10: \quad e^{A}=(e^a,e^{a'}), \tag{4.21}$$

where $e^a=dx^m e_m{}^a(x)$ $(a=0,1,2,3)$ and x^m $(m=0,1,2,3)$ are AdS_4 vielbeins and coordinates respectively, $e^{\hat{a}'}$ are the S^7 vielbeins (4.12) and $e^{a'}=dy^{n'}e_{n'}{}^{a'}(y)$ are the CP^3 vielbeins.

For further comparison with the superspace case, it is important to note that in the given realization, the components $e_{\hat{B}}{}^{\hat{A}}(x,y)$ of the $D=11$ vielbeins of $AdS_4\times S^7$ do not depend on the U(1) bundle coordinate z and that their components $e_7{}^a$ and $e_7{}^{a'}$ vanish

$$e_7{}^a=0, \qquad e_7{}^{a'}=0\,. \tag{4.22}$$

Such a choice of the vielbein directly corresponds to the Kaluza-Klein ansatz for the compactification on a circle S^1 and $A_{m'}(y)$ is associated with the potential of an Abelian gauge field in the reduced theory.

In our case the field strength of $A_{m'}(y)$ is the flux proportional to the Kähler form $J_{a'b'}$ on CP^3

$$dA=F_2=\frac{1}{2}\,e^{a'}\,e^{b'}F_{b'a'}=e^{a'}\,e^{b'}\,J_{a'b'}\,. \tag{4.23}$$

Together with the F_4 flux whose non-zero components are along AdS_4, with $F_{abcd}=-6\,\varepsilon_{abcd}$, the F_2 flux completes (the bosonic part of) the compactification of IIA type supergravity on $AdS_4\times CP^3$.

It should be noted that the Kaluza-Klein condition analogous to (4.22) is always required in order for the action and equations of motion of the dimensionally reduced theory to have a conventional gauge structure, describing the interactions of an Abelian gauge field with gravity. In general, it can always be achieved by performing an appropriate local Lorentz transformation of the vielbeins in the original $D+1$-dimensional theory such that their components with one world index along the compactified direction and D indices along the reduced D-dimensional tangent space vanish (as in eq. (4.22)).

In the case of the Kaluza-Klein dimensional reduction of the bosonic space $AdS_4\times S^7$ to ten dimensions, we have arrived at the Kaluza-Klein ansatz corresponding to the representation of the S^7 as a Hopf fibration over CP^3. As we shall see, this is not the case when the Hopf fibration is lifted to the D=11 supermanifold $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ having 32 fermionic directions. An additional local Lorentz transformation, which is *not* part of the

[16] We put the radius of S^7 and the corresponding size of CP^3 to be one. The AdS_4 radius of the $D=11$ and $D=10$ solution is 1/2 of that of the compact manifold.

OSp(8|4) isometries, will be required to bring the supervielbeins of this supermanifold to the Kaluza-Klein form, thus allowing us to perform its dimensional reduction to the $AdS_4 \times CP^3$ type IIA supergravity solution in $D = 10$ superspace with 32 fermionic coordinates.

5 Lifting the S^7 Hopf fibration to $D = 11$ superspace

The superfield descriptions of type IIA $D = 10$ and of $D = 11$ supergravity are based on a superspace with 32 fermionic coordinates which in the $AdS_4 \times CP^3$ and $AdS_4 \times S^7$ backgrounds can be described by spinors $\theta^{\alpha\alpha'}$ carrying AdS_4 Majorana spinor indices ($\alpha = 1, 2, 3, 4$) and the indices ($\alpha' = 1, \ldots, 8$) of an 8-dimensional spinor representation of SO(6) or SO(8), respectively. In the $AdS_4 \times S^7$ solution of $D = 11$ supergravity, $\theta^{\alpha\alpha'}$ are the coordinates of the coset supermanifold OSp(8|4)/SO(7) $\times$ SO(1,3) associated with the 32 Grassmann-odd generators $Q_{\alpha\alpha'}$ of OSp(8|4) (for a detailed description see e.g. [24, 25]).

On the other hand, the coset supermanifold OSp(6|4)/U(3) $\times$ SO(1,3) (for its detailed description see e.g. [27–29, 31]) is parametrized by ten bosonic coordinates $X^M = (x^m,\, y^{m'})$ corresponding to its bosonic body $AdS_4 \times CP^3$ and by 24 fermionic coordinates $\vartheta^{\alpha a'}$, where again $\alpha = 1, 2, 3, 4$ are the AdS_4 Majorana spinor indices and $a' = 1, \ldots, 6$ are the indices of a 6-dimensional representation of SO(6) $\simeq$ SU(4). The 24 fermionic coordinates are associated with the 24 Grassmann-odd generators $Q_{\alpha a'}$ of the OSp(6|4) algebra.

The 24 generators $Q_{\alpha a'}$ and the corresponding fermionic coordinates $\vartheta^{\alpha a'}$ can be obtained from the 32 Grassmann-odd generators $Q_{\alpha\alpha'}$ of OSp(8|4) and the coordinates $\theta^{\alpha\alpha'}$ by acting on the SO(8) spinor indices with the projection matrix $\mathcal{P}_6$ introduced in [40] (see [31] and appendices B and C.2 for more details)

$$\mathcal{P}_6 = \frac{1}{8}(6 - J)\,, \tag{5.1}$$

where J is the 8×8 symmetric matrix

$$J = -iJ_{a'b'}\,\gamma^{a'b'}\,\gamma^7 \qquad \text{such that} \qquad J^2 = 4J + 12\,, \tag{5.2}$$

with $\gamma^{a'}_{\alpha'\beta'}$ ($a' = 1, \ldots, 6$) and $\gamma^7_{\alpha'\beta'}$ being seven 8×8 gamma matrices (see appendix A).

The matrix J has six eigenvalues -2 and two eigenvalues 6, i.e. its diagonalization is given by

$$J = \mathrm{diag}(-2, -2, -2, -2, -2, -2, 6, 6)\,. \tag{5.3}$$

Therefore, the projector (5.1) when acting on an 8-dimensional spinor annihilates 2 components and preserves 6 of its components, while the complementary projector

$$\mathcal{P}_2 = \frac{1}{8}(2 + J)\,, \qquad \mathcal{P}_2 + \mathcal{P}_6 = \mathbf{1} \tag{5.4}$$

annihilates 6 and preserves 2 spinor components.

Thus the generators

$$(\mathcal{P}_6\, Q)_{\alpha\alpha'} \qquad \Longleftrightarrow \qquad Q_{\alpha a'}\,, \qquad a' = 1, \ldots, 6 \tag{5.5}$$

have 24 non-zero components and can be associated with the 24 Grassmann-odd generators $Q_{\alpha a'}$ of OSp(6|4). Accordingly, the 24 fermionic variables

$$(\mathcal{P}_6\,\theta)^{\alpha\alpha'} \qquad \Longleftrightarrow \qquad \vartheta^{\alpha a'}\,, \qquad a' = 1,\ldots,6 \tag{5.6}$$

can be associated with the 24 fermionic coordinates $\vartheta^{\alpha a'}$ of OSp(6|4)/U(3) × SO(1, 3).

On the other hand, acting on $Q_{\alpha\alpha'}$ with the projector $\mathcal{P}_2$ (5.4) one gets 8 generators

$$(\mathcal{P}_2\,Q)_{\alpha\alpha'} \qquad \Longleftrightarrow \qquad \mathcal{Q}_{\alpha i}, \qquad i = 7,8 \tag{5.7}$$

which correspond to the eight supersymmetries broken by the $AdS_4 \times CP^3$ background. The associated 8 fermionic coordinates of the type IIA superspace are

$$(\mathcal{P}_2\,\theta)^{\alpha\alpha'} \qquad \Longleftrightarrow \qquad \upsilon^{\alpha i}\,, \qquad i = 7,8\,. \tag{5.8}$$

Note that the eight operators $\mathcal{Q}_{\alpha i}$ generate an OSp(2|4) subalgebra of OSp(8|4) (see appendices B and C.2 for more details)

$$\{\mathcal{Q}_{\alpha i},\mathcal{Q}_{\beta j}\} = -2i\epsilon_{ij}\,\gamma^5_{\alpha\beta}\,T_2 - 2\,\delta_{ij}\,(\gamma^a_{\alpha\beta}\,P_a - i(\gamma^5\gamma^{ab})_{\alpha\beta}\,M_{ab}), \tag{5.9}$$
$$[M_{ab},\mathcal{Q}_{\alpha i}] = -\frac{1}{2}\,\mathcal{Q}_{\beta i}\,(\gamma_{ab})^{\beta}{}_{\alpha}\,, \quad [P_a,\mathcal{Q}_{\alpha i}] = i\,\mathcal{Q}_{\beta i}\,(\gamma^5\gamma_a)^{\beta}{}_{\alpha}\,, \quad [T_2,\mathcal{Q}_{\alpha i}] = 2\epsilon_{ij}\,\mathcal{Q}_{\alpha j},$$

where T_2 is the U(1) ≃ SO(2) generator of SO(8) in OSp(8|4) which commutes with OSp(6|4), so that OSp(6|4) × SO(2) is a subgroup of OSp(8|4). Recall that we have introduced the generator T_2 in section 4.

The generators P_a and M_{ab} form the Sp(4) ≃ Spin(2, 3) algebra

$$[P_a,P_b] = -4M_{ab}, \qquad [M_{ab},M_{cd}] = \eta_{ac}\,M_{bd} + \eta_{bd}\,M_{ac} - \eta_{bc}\,M_{ad} - \eta_{ad}\,M_{bc}\,. \tag{5.10}$$
$$[M_{ab},P_c] = \eta_{ac}\,P_b - \eta_{bc}\,P_a\,. \tag{5.11}$$

5.1 Hopf fibration of the OSp(6|4)/U(3) × SO(1, 3) supercoset

Let us now lift the OSp(6|4)/U(3) × SO(1, 3) solution of IIA supergravity to $D = 11$ by constructing a U(1) bundle over this supermanifold along the lines of the Hopf fibration of S^7 discussed in section 4. This is realized by constructing a coset superspace

$$\frac{\mathrm{OSp}(6|4)\times\mathrm{U}(1)}{\mathrm{SU}(3)\times U'(1)\times\mathrm{SO}(1,3)}\,, \tag{5.12}$$

having 11 bosonic and 24 fermionic directions. In (5.12) U(1) is generated by T_2 and $U'(1)$ is generated by $T' = \frac{3}{4}(T_2 - T_1)$ (see eq. (4.1)). We take a coset representative of this superspace in the following form

$$K_{11,24}(x,y,z,\vartheta) = K_{10,24}(x,y,\vartheta)\,e^{z\,T_2} \tag{5.13}$$

where $K_{10,24}(x,y,\vartheta)$ is a coset representative of OSp(6|4)/U(3) × SO(1, 3) which can be taken in any convenient form, e.g. in one of those considered in [27–29, 31] (or appendix A).

The supervielbeins and superconnections of the supercoset (5.12) are encoded in the OSp(6|4) $\times$ U(1) Cartan form

$$K^{-1}_{11,24}\, d\, K_{11,24} = K^{-1}_{10,24}\, d\, K_{10,24} + dz\, T_2\,, \tag{5.14}$$

where the OSp(6|4) Cartan form

$$\begin{aligned} K^{-1}_{10,24}\, d\, K_{10,24} \;=\;& E^a(x,y,\vartheta)\, P_a + E^{a'}(x,y,\vartheta)\, P_{a'} + E^{\alpha a'}(x,y,\vartheta)\, Q_{\alpha a'} \\ &+\frac{1}{2}\Omega^{ab}(x,y,\vartheta)\, M_{ab} + \Omega^I(x,y,\vartheta)\, L_I + A(x,y,\vartheta)\, T_1 \end{aligned} \tag{5.15}$$

contains the supervielbeins and superconnections of OSp(6|4)/U(3) $\times$ SO(1,3) whose explicit form can be found in [27–29, 31] (or appendix A). The SU(3) $\times$ U(1) generators L_I and T_1 have been introduced in section 4.

Now, as in the case of the 7-sphere, eq. (4.11), we single out proper supervielbeins and superconnections of the supercoset (5.12) as follows

$$\begin{aligned} K^{-1}_{11,24} dK_{11,24} \;=\;& E^a(x,y,\vartheta) P_a + E^{a'}(x,y,\vartheta) P_{a'} + (dz + A(x,y,\vartheta))\, P_7 + E^{\alpha a'}(x,y,\vartheta) Q_{\alpha a'} \\ &+\frac{1}{2}\Omega^{ab}(x,y,\vartheta) M_{ab} + \Omega^I(x,y,\vartheta) L_I + (dz - \frac{1}{3}A(x,y,\vartheta))T'. \end{aligned} \tag{5.16}$$

Given that

$$Z^{\tilde{\mathcal{M}}} = (x^m, y^{m'}, \vartheta^{\alpha a'}) \tag{5.17}$$

are the supercoordinates parametrizing OSp(6|4)/U(3) $\times$ SO(1,3) and that z is the coordinate of the Hopf fiber, the 11 bosonic and 24 fermionic supervielbeins are given by

$$\begin{aligned} &E^{\hat{A}} = (E^a,\, E^{\hat{a}'})\,, \quad E^a = dZ^{\tilde{\mathcal{M}}}\, E_{\tilde{\mathcal{M}}}{}^a(x,y,\vartheta)\,, \quad E^{\hat{a}'} = dZ^{\tilde{\mathcal{M}}}\, E_{\tilde{\mathcal{M}}}{}^{\hat{a}'}(x,y,\vartheta) = (E^{a'}, E^7)\,, \\ &E^7 = dz + dZ^{\tilde{\mathcal{M}}}\, A_{\tilde{\mathcal{M}}}(x,y,\vartheta) \end{aligned} \tag{5.18}$$

while the 24 fermionic supervielbeins are

$$E^{\alpha a'} = dZ^{\tilde{\mathcal{M}}}\, E_{\tilde{\mathcal{M}}}{}^{\alpha a'}(x,y,\vartheta)\,. \tag{5.19}$$

The connections of the stability group U(3) $\times$ SO(1,3) are given in the last line of (5.16).

We see that the components of the supervielbeins and connections do not depend on the 11th coordinate z, which appears only in the differential of the E^7 vielbein. Moreover, the supervielbein components $dz\, E_7^{\mathcal{A}} = (dz\, E_7{}^a,\, dz\, E_7{}^{a'},\, dz\, E_7{}^{\alpha a'})$ are all zero. Thus, the realization of the coset supermanifold (5.12) considered above has a Hopf fibration structure generalizing that of the 7-sphere. The dimensional reduction of this supermanifold to $D = 10$ is then straightforward. One must just project it orthogonally to the U(1) fiber direction, i.e. to pick E^a, $E^{a'}$ and $E^{\alpha a'}$ as the supervielbeins of the $D = 10$ superspace and to consider $dZ^{\tilde{\mathcal{M}}}\, A_{\tilde{\mathcal{M}}}(x,y,\vartheta)$ as the RR one-form potential of the type IIA supergravity theory. Note that in this reduced type IIA superspace solution, the dilaton superfield is constant and the dilatino superfield vanishes.

The difference with respect to the purely bosonic case is that whereas the S^7 fibration has an enhanced SO(8) isometry, the isometry supergroup of the supermanifold (5.12) is still OSp(6|4) × U(1), since SO(8) is not its subgroup. The extension to SO(8) and, hence, to OSp(8|4) requires the introduction of 8 additional Grassmann-odd generators.

On the other hand, it can be directly verified that the $D = 11$ superspace with 24 fermionic directions considered above is a solution of superfield constraints of $D = 11$ supergravity (and, hence, of its equations of motion). It thus provides a description of the maximally supersymmetric $AdS_4 \times S^7$ solution in a reduced superspace which can be regarded as a sub-superspace of OSp(8|4)/SO(7) × SO(1, 3).

5.2 U(1) bundle structure of the OSp(8|4)/SO(7) × SO(1, 3) supercoset

Let us now extend the supercoset (5.12) to the full OSp(8|4)/SO(7) × SO(1, 3) supercoset. This is achieved by taking the following group element of OSp(8|4) as the coset representative of OSp(8|4)/SO(7) × SO(1, 3)

$$K_{11,32}(x, y, z, \theta) = K_{11,24}(x, y, z, \vartheta)\, e^{\upsilon^{\alpha i}\, \mathcal{Q}_{\alpha i}} = K_{10,24}(x, y, \vartheta)\, e^{z\, T_2}\; e^{\upsilon^{\alpha i}\, \mathcal{Q}_{\alpha i}}\,, \tag{5.20}$$

where $K_{11,24}(x, y, z, \vartheta)$ is the same coset representative as in (5.13) and $\theta = (\vartheta, \upsilon)$ are the 32-component fermionic coordinates which, using the projectors (5.1) and (5.4), split into 24-component ϑ's and 8-component υ's. Note that the group element $e^{\upsilon^{\alpha i}\, \mathcal{Q}_{\alpha i}}$ can be regarded as the representative of the purely fermionic supercoset $\frac{\mathrm{OSp}(2|4)}{\mathrm{SO}(2)\times \mathrm{SO}(2,3)}$.

The OSp(8|4)-valued Cartan form constructed with (5.20) is

$$\begin{aligned} K^{-1}_{11,32}\, dK_{11,32} &= e^{-\upsilon\, \mathcal{Q}}\, (K^{-1}_{11,24}\, d\, K_{11,24})\, e^{\upsilon\, \mathcal{Q}} + e^{-\upsilon\, \mathcal{Q}}\, d\, e^{\upsilon\, \mathcal{Q}} \\ &= e^{-\upsilon\, \mathcal{Q}}\, (K^{-1}_{10,24}\, d\, K_{10,24})\, e^{\upsilon\, \mathcal{Q}} + dz\, e^{-\upsilon\, \mathcal{Q}}\, T_2\, e^{\upsilon\, \mathcal{Q}} + e^{-\upsilon\, \mathcal{Q}}\, d\, e^{\upsilon\, \mathcal{Q}} \end{aligned} \tag{5.21}$$

or, using the commutation relations (5.9) and the form of $K^{-1}_{10,24}\, d\, K_{10,24}$ (5.15)

$$\begin{aligned} K^{-1}_{11,32}\, dK_{11,32} = {}& E^{a}(x, y, \vartheta)\, e^{-\upsilon\, \mathcal{Q}}\, P_a\, e^{\upsilon\, \mathcal{Q}} + E^{\alpha a'}(x, y, \vartheta)\, e^{-\upsilon\, \mathcal{Q}}\, Q_{\alpha a'}\, e^{\upsilon\, \mathcal{Q}} + E^{a'}(x, y, \vartheta)\, P_{a'} \\ & + \frac{1}{2}\Omega^{ab}(x, y, \vartheta)\, e^{-\upsilon\, \mathcal{Q}}\, M_{ab}\, e^{\upsilon\, \mathcal{Q}} + \Omega^{I}(x, y, \vartheta)\, L_I + A(x, y, \vartheta)\, T_1 \\ & + dz\, e^{-\upsilon\, \mathcal{Q}}\, T_2\, e^{\upsilon\, \mathcal{Q}} + e^{-\upsilon\, \mathcal{Q}}\, d\, e^{\upsilon\, \mathcal{Q}}\,. \end{aligned} \tag{5.22}$$

Note that the supervielbein and connection terms in (5.22) corresponding to the SU(4) generators $P_{a'}$, L_I and T_1 do not receive contributions from $\upsilon^{\alpha i}$, since these generators commute with $\mathcal{Q}_{\alpha i}$.

Furthermore, we can expand the Cartan form (5.22) in the basis of the OSp(8|4) generators. The expansion contains generators along the AdS_4, CP^3 and z directions, along the generators of their stability group SO(1, 3) × SU(3) × $U'(1)$ and the rest. It is given by

$$\begin{aligned} K^{-1}_{11,32}\, dK_{11,32} = {}& E^{a}_{11,32}\, P_a + E^{a'}_{11,32}\, P_{a'} + E^{7}_{11,32}\, P_7 + E^{\alpha i}_{11,32}\, \mathcal{Q}_{\alpha i} + E^{\alpha a'}_{11,32}\, Q_{\alpha a'} \\ & + \frac{1}{2}\Omega^{ab}_{11,32}\, M_{ab} + \Omega^{I}_{10,24}\, L_I + \Omega'_{11,32}\, T' + \tilde{\Omega}^{a' i}_{11,32}\, \tilde{M}_{a' i}, \end{aligned} \tag{5.23}$$

where, we remind the reader that P_7 and T' were defined in (4.1) and (4.2), while

$$\tilde{M}_{a' i} \quad \Leftrightarrow \quad \frac{1}{4}\, \mathcal{P}_6 \gamma^{\tilde{a}' \tilde{b}'} \mathcal{P}_2\, M_{\tilde{a}' \tilde{b}'} + \frac{i}{2}\, \mathcal{P}_6 \gamma^{a'} \mathcal{P}_2\; P_{a'}, \tag{5.24}$$

with $M_{\tilde{a}'\tilde{b}'}$ being the generators of SO(8) (see appendix B). $\tilde{M}_{a'i}$ are the generators which complete the $\mathrm{SO}(6)\times\mathrm{SO}(2)$ algebra to SO(8). They differ from the generators $M_{a'i}$ introduced in appendix B, eqs. (B.21)–(B.23), by the shift along the CP^3 translations generated by $P_{a'} = -M_{a'8} + J_{a'}{}^{b'} M_{b'7}$. The reason for this redefinition is that the commutators of the generators $M_{a'i}$, defined in eqs. (B.22), produces the generators of the $\mathrm{SO}(6)\times\mathrm{SO}(2)$ subgroup of the SO(8) group, and, in particular the CP^3 coset generators $P_{a'}$. Thus, $M_{a'i}$ themselves cannot be regarded as generators belonging to the structure group SO(7) of the 7-sphere. The commutators of the SO(7) generators should not produce the translations along S^7. Therefore, to make $M_{a'i}$ part of SO(7) one must redefine them as in eq. (5.24). This redefinition results in the appearance of the additional (second) term in the expression for the supervielbein $E^{a'}_{11,32}$ in eq. (5.25) below.

All functions of $\upsilon^{\alpha i}$ in (5.23) can be explicitly computed using the commutation relations (5.9), (B.21) and (B.23) and applying the method described e.g. in [22–25]. The supervielbeins we get are

$$
\begin{aligned}
E^{a}_{11,32} &= E^{a}_{10,24} - 4\upsilon\gamma^a \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\, D\upsilon \,+ dz\, E_7{}^a(\upsilon)\,,\\
E^{a'}_{11,32} &= E^{a'}_{10,24} - 2\upsilon\, \frac{\sinh m}{m}\gamma^{a'}\gamma^5\, E_{10,24}\,,\\
E^{7}_{11,32} &= dz\,\Phi(\upsilon) + A_{10,24} - 4i\upsilon\,\varepsilon\gamma^5\, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\, D\upsilon\,,\\
E^{\alpha i}_{11,32} &= \left(\frac{\sinh\mathcal{M}}{\mathcal{M}}\,(D\upsilon - 2dz\,\varepsilon\upsilon)\right)^{\alpha i},\\
E^{\alpha a'}_{11,32} &= E^{\alpha a'}_{10,24} - 8E^{\beta a'}_{10,24}\left(i\gamma^5\,\upsilon\,\frac{\sinh^2 m/2}{m^2}\right)_{\beta i}\upsilon^{\alpha i}\,,
\end{aligned}
\tag{5.25}
$$

the SO(1, 3) connection is

$$
\Omega^{ab}_{11,32} = \Omega^{ab}_{10,24} + 8i\upsilon\gamma^{ab}\gamma^5\,\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\,(D\upsilon - 2dz\,\varepsilon\upsilon)\,,
\tag{5.26}
$$

the one-form $\tilde{\Omega}^{a'i}$ is

$$
\tilde{\Omega}^{a'i}_{11,32} = 4E^{\alpha a'}_{10,24}\left(i\gamma^5\,\upsilon\,\frac{\sinh m}{m}\right)^{i}_{\alpha}
\tag{5.27}
$$

and the one-form $\Omega'_{11,32}$ is

$$
\Omega'_{11,32} = dz\,\Phi(\upsilon) - \frac{1}{3}\,A_{10,24} - 4i\upsilon\,\varepsilon\gamma^5\,\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\,D\upsilon\,.
\tag{5.28}
$$

The SO(7) connection in the considered realization of the supercoset $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ can be computed from (5.23) and has the form

$$
\begin{aligned}
\frac{1}{2}\,\Omega^{a'b'}_{11,32}\,M_{a'b'} + \Omega^{a'7}_{11,32}\,M_{a'7} = {}&\left(E^{b'}_{11,32} + 4\upsilon\,\frac{\sinh m}{m}\gamma^{b'}\gamma^5\,E_{10,24}\right)J_{b'}{}^{a'}\,M_{a'7}\\
&+\frac{1}{2}\left(\Omega^{a'b'}_{10,24} - E^{7}_{11,32}\,J^{a'b'} - 2i\upsilon\,\frac{\sinh m}{m}\gamma^{a'b'}\gamma^5\,E_{10,24}\right)M_{a'b'}.
\end{aligned}
\tag{5.29}
$$

The functions appearing in (5.25)–(5.28) are defined as[17]

$$
\begin{aligned}
(\mathcal{M}^2)^{\alpha i}{}_{\beta j} &= 4i(\varepsilon\upsilon)^{\alpha i}(\upsilon\varepsilon\gamma^5)_{\beta j} - 2i(\gamma^5\gamma^a\upsilon)^{\alpha i}(\upsilon\gamma_a)_{\beta j} - i(\gamma^{ab}\upsilon)^{\alpha i}(\upsilon\gamma_{ab}\gamma^5)_{\beta j}\,,\\
(m^2)^{ij} &= -4i\upsilon^i\,\gamma^5\,\upsilon^j\,,
\end{aligned}
\tag{5.30}
$$

$$
\begin{aligned}
E_7{}^a(\upsilon) &= 8\,\upsilon\gamma^a\,\frac{\sinh^2\mathcal{M}/2}{\mathcal{M}^2}\,\varepsilon\,\upsilon\,,\\
\Phi(\upsilon) &= 1 + 8i\,\upsilon\,\varepsilon\gamma^5\,\frac{\sinh^2\mathcal{M}/2}{\mathcal{M}^2}\,\varepsilon\upsilon
\end{aligned}
\tag{5.31}
$$

and

$$
D\upsilon = \left(d + iE^a_{10,24}\gamma^5\gamma_a - \frac{1}{4}\Omega^{ab}_{10,24}\gamma_{ab}\right)\upsilon\,. \tag{5.32}
$$

All quantities in (5.25)–(5.32) labeled as $E_{10,24}$, $\Omega_{10,24}$ *etc.* are the ones which describe the supercoset $OSp(6|4)/U(3)\times SO(1,3)$ and are explicitly known (see e.g. [22, 27–29, 31] and appendix A).

Analyzing eqs. (5.22)–(5.32) we observe, in particular, that due to the multiplication by $e^{\upsilon Q}$ in (5.22), the AdS_4 supervielbeins and the $SO(1,3)$ superconnections (5.15) corresponding to the supercoset $OSp(6|4)/U(3)\times SO(1,3)$ acquire non-trivial dependence on the 8 additional fermionic variables $\upsilon^{\alpha i}$. In the first line of (5.22) and in (5.23) there are also terms with components of the superconnection corresponding to the generators (5.24) which extend the $SO(6)\times SO(2)$ algebra to $SO(8)$ because of the non-trivial anti-commutators of the 24 supersymmetry generators $Q_{\alpha a'}$ with the 8 supersymmetry generators $\mathcal{Q}_{\alpha i}$ (eqs. (B.21)–(B.23)).

We also observe that, in contrast to the cases discussed in sections 4 and 5.1, the $U(1)$-bundle realization of the $OSp(8|4)/SO(7)\times SO(1,3)$ supercoset geometry in (5.23) does not allow for its direct dimensional reduction to a $D=10$ superspace because of the presence of the term $dz\,E_7{}^a(\upsilon)$. This term contributes to the components of the supervielbein along the directions tangent to AdS_4 and has a 'leg' along the compactified direction parametrized by the z-coordinate.[18] As we discussed in the end of section 4, to perform the Kaluza-Klein dimensional reduction such components of the (super)vielbein must be put to zero.

From the supervielbeins in (5.25) we can also construct the supergeometry corresponding to the superspace with $AdS_4\times S^7/Z_k$ bosonic body, a background of eleven dimensional supergravity which preserves 24 supersymmetries (for $k>2$) and is the near horizon geometry of N M2-branes probing the C^4/Z_k singularity. Geometrically, this superspace is obtained by orbifolding the $OSp(8|4)/SO(7)\times SO(1,3)$ supercoset geometry by $Z_k\subset U(1)$, where $U(1)$ is the commutant of $SU(4)$ in $SO(8)$. The corresponding supervielbeins are simply obtained from those in (5.25) by replacing $z\to z/k$.

[17] Note that only positive even powers of $\mathcal{M}$ and m appear in the above expressions when they are expanded.

[18] A somewhat amusing remark is that the term $dz\,E_7{}^a(\upsilon)$, in a certain sense, 'mixes' the AdS_4 geometry with the $U(1)$ fiber direction of the S^7. On the other hand, the more 'natural' terms like $dz\,E_7{}^{a'}(\upsilon)$ along the CP^3 tangent space, which would mix the Hopf fiber direction with CP^3, are absent. They would correspond to some vielbein components on the S^7.

JHEP03(2009)015

5.3 Hopf fibration form of the $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ geometry and its reduction to type IIA superspace

To eliminate the term $dz\,E_7{}^a(\upsilon)$ from the $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ supervielbein we should perform an appropriate local Lorentz rotation in the 5-plane (E^a, E^7) tangential to $AdS_4\times S^1$, where S^1 is the U(1) fiber direction in S^7. Obviously, such a transformation is not an isometry of the coset supermanifold $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ and should therefore be regarded simply as a change of local frame. Upon this Lorentz transformation we shall get the $D=11$ supervielbeins in a form which will allow us to directly identify the corresponding $D=10$ supervielbeins, the RR one-form gauge superfield and the dilaton superfield of type IIA supergravity.

Let $E^{\hat A} = (E^a,\, E^{a'},\, E^7)$ be the 11 bosonic components of the $\mathrm{OSp}(8|4)/\mathrm{SO}(7)\times\mathrm{SO}(1,3)$ supervielbein given in (5.25). To eliminate the $dz\,E_7{}^a(\upsilon)$ component of E^a we perform the following Lorentz transformation

$$\underline{\mathcal{E}}^a = E^b\,\Lambda_b{}^a(\upsilon) + E^7\,\Lambda_7{}^a(\upsilon)\,, \qquad \underline{\mathcal{E}}^7 = E^b\,\Lambda_b{}^7(\upsilon) + E^7\,\Lambda_7{}^7(\upsilon)\,, \tag{5.33}$$

where the parameters $\Lambda_{\hat b}{}^{\hat a}(\upsilon)$ ($\hat a = (a,7) = 0,1,2,3,7$) depend on the 8 fermionic coordinates $\upsilon^{\alpha i}$ and satisfy the 5-dimensional Lorentz group orthogonality conditions

$$\Lambda_{\hat a}{}^{\hat c}\,\Lambda_{\hat b}{}^{\hat d}\,\eta_{\hat c\hat d} = \eta_{\hat a\hat b}, \tag{5.34}$$

or in components

$$\Lambda_a{}^c\,\Lambda_b{}^d\,\eta_{cd} + \Lambda_a{}^7\,\Lambda_b{}^7 = \eta_{ab}, \quad \Lambda_7{}^c\,\Lambda_7{}^d\,\eta_{cd} + (\Lambda_7{}^7)^2 = 1, \quad \Lambda_7{}^c\,\Lambda_a{}^d\,\eta_{cd} + \Lambda_7{}^7\Lambda_a{}^7 = 0 \tag{5.35}$$

and

$$\Lambda_a{}^c\,\Lambda_b{}^d\,\eta^{ab} + \Lambda_7{}^c\,\Lambda_7{}^d = \eta^{cd}, \quad \Lambda_c{}^7\,\Lambda_d{}^7\,\eta^{cd} + (\Lambda_7{}^7)^2 = 1, \quad \Lambda_c{}^7\,\Lambda_d{}^a\,\eta^{cd} + \Lambda_7{}^7\Lambda_7{}^a = 0\,. \tag{5.36}$$

In addition $\Lambda_{\hat b}{}^{\hat a}(\upsilon)$ is determined by the requirement that the $\underline{\mathcal{E}}_7{}^a$ component of the transformed supervielbein vanishes and that at $\upsilon = 0$ it reduces to the unit matrix

$$\Lambda_{\hat b}{}^{\hat a}(\upsilon)|_{\upsilon=0} = \delta_{\hat b}{}^{\hat a}\,, \qquad \underline{\mathcal{E}}_7{}^a = E_7{}^b\,\Lambda_b{}^a + \Phi\,\Lambda_7{}^a = 0\,, \tag{5.37}$$

where $\Phi(\upsilon) := E_7{}^7$, $\Phi(0) = 1$ (see eq. (5.31)). From eq. (5.37) we find that

$$\Lambda_7{}^a(\upsilon) = -\frac{1}{\Phi(\upsilon)}\,E_7{}^b(\upsilon)\,\Lambda_b{}^a(\upsilon)\,. \tag{5.38}$$

Then, solving the orthogonality conditions (5.35) and (5.36) we find the expressions for the parameters of the Lorentz transformation in terms of $E_7{}^a(\upsilon)$ and $\Phi(\upsilon)$

$$\Lambda_7{}^7 = \frac{\Phi}{\sqrt{\Phi^2 + E^2}}\,, \tag{5.39}$$

$$\Lambda_a{}^7 = \frac{E_{7\,a}}{\sqrt{\Phi^2 + E^2}}\,, \tag{5.40}$$

where $E^2 \equiv E_7{}^a\, E_7{}^b\, \eta_{ab}$, and

$$\Lambda_a{}^c\,\Lambda_b{}^d\,\eta_{cd} = \eta_{ab} - \frac{E_{7\,a}\,E_{7\,b}}{\Phi^2+E^2}\,, \qquad (5.41)$$
$$\Lambda_a{}^b = \delta_a{}^b - E_{7a}\,E_7{}^b\,\frac{\sqrt{\Phi^2+E^2}-\Phi}{E^2\,\sqrt{\Phi^2+E^2}} \Rightarrow \det\Lambda_a{}^b = \frac{\Phi}{\sqrt{\Phi^2+E^2}}\,.$$

Finally eq. (5.38) can be rewritten as

$$\Lambda_7{}^a = -\frac{E_7{}^a}{\sqrt{\Phi^2+E^2}}\,. \qquad (5.42)$$

One can notice that $\Lambda_{\hat b}{}^{\hat a}$ depend only on the vector parameter $\frac{1}{\Phi}\,E_7{}^a$ and thus can be regarded as a kind of "Lorentz boost" along the S^7 fiber direction.

The following ten components of the Lorentz transformed $D = 11$ supervielbeins

$$\underline{\mathcal{E}}^A(x,y,\vartheta,\upsilon) = (\underline{\mathcal{E}}^a,\,\underline{\mathcal{E}}^{a'})\,, \qquad A = 0,1,\ldots,9;\quad a = 0,1,2,3; a' \;= 1,\ldots,6 \qquad (5.43)$$
$$\underline{\mathcal{E}}^a = E^b\,\Lambda_b{}^a(\upsilon) + E^7\,\Lambda_7{}^a(\upsilon)\,,\quad \underline{\mathcal{E}}^{a'} = E^{a'}$$

form an appropriate bosonic supervielbein of the complete $(32\text{-}\theta)$ superfield solution of type IIA supergravity corresponding to the $AdS_4\times CP^3$ vacuum. The IIA dilaton superfield is

$$e^{\frac{2}{3}\phi(\upsilon)} = \Phi\,\Lambda_7{}^7 + E_7{}^a\,\Lambda_a{}^7 = \sqrt{\Phi^2 + E_7{}^a\,E_7{}^b\,\eta_{ab}}\,. \qquad (5.44)$$

One can notice that the dilaton superfield of this type IIA solution depends only on the eight fermionic coordinates $\upsilon^{\alpha i}$ which correspond to the broken supersymmetries of the $AdS_4\times CP^3$ background.

In addition to the Lorentz rotation of the vector supervielbeins, we should also perform a corresponding Lorentz rotation of the components of the connections and of the spinor supervielbeins $E^{\alpha a'}$ and $E^{\alpha i}$. In particular, the Lorentz rotation of the connection components $\Omega^{a'7}$ will produce a "mixed" AdS_4–CP^3 term $\Omega^{a'a} = \Omega^{a'7}\Lambda_7{}^a$ which transforms as a tensor under $\mathrm{U}(3)\times\mathrm{SO}(1,3)$ and hence can be absorbed into a redefined torsion of the type IIA superspace.

As far as the Lorentz rotation of the spinor supervielbeins is concerned, it is worth noting that the Lorentz rotation of the spinors associated with (5.34)–(5.36) is generated by the gamma-matrices $\Gamma^a\Gamma^{11} = \gamma^a\,\gamma^5\otimes\gamma^7$ which commute with the projectors (5.1) and (5.4) and thus does not mix the 24 and 8-component spinors. The explicit form of the Lorentz rotation acting on spinors, $S_{\underline{\alpha}}{}^{\underline{\beta}}(\upsilon)$ ($\underline{\alpha} = (\alpha\alpha')$), can be derived using the well known relations between the vector and spinor representations of the Lorentz group

$$S^{-1}\,\Gamma^{\hat a}\,S = \Gamma^{\hat b}\,\Lambda_{\hat b}{}^{\hat a}\,,\qquad S_{\underline{\alpha}}{}^{\underline{\gamma}}\;S_{\underline{\beta}}{}^{\underline{\delta}}\;\mathcal{C}_{\underline{\gamma}\underline{\delta}} = \mathcal{C}_{\underline{\alpha}\underline{\beta}}\,, \qquad (5.45)$$

where $\Gamma^{\hat a} = (\Gamma^a,\Gamma^{11})$ are 32×32 gamma-matrices defined in (A.8) and $\mathcal{C} = C\otimes C'$ is the corresponding charge conjugation matrix.

Since the Lorentz transformation giving rise to supervielbeins and connections compatible with the KK ansatz corresponds to a Lorentz rotation with the "velocity" parameter $\mathrm{w}^a = E_7{}^a/\Phi$, the corresponding matrix acting on the fermions (5.45) is given by

$$
\begin{aligned}
S &= \exp\left(-\frac{1}{2}\frac{\mathrm{w}^a}{|\mathrm{w}|}\Gamma_a\Gamma_{11}\,\tan^{-1}|\mathrm{w}|\right) \qquad\qquad (\mathrm{w}^a = E_7{}^a/\Phi) \\
&= 2^{-1/2}(1+\mathrm{w}^2)^{-1/4}\left(\sqrt{\sqrt{1+\mathrm{w}^2}+1} - \frac{\mathrm{w}^a}{|\mathrm{w}|}\Gamma_a\Gamma_{11}\sqrt{\sqrt{1+\mathrm{w}^2}-1}\right).
\end{aligned} \tag{5.46}
$$

Performing the Lorentz rotation described above, the $D=11$ supervielbeins (upon a Weyl rescaling) acquire a form which is suitable for the dimensional reduction to $D=10$ superspace in the string frame [46, 47]:

$$
\begin{aligned}
\underline{\mathcal{E}}^{\hat{A}} &= (e^{-\frac{1}{3}\phi}\,\mathcal{E}^A,\,\underline{\mathcal{E}}^{11}), \qquad \underline{\mathcal{E}}^{11} = e^{\frac{2}{3}\phi}\,(dz+\mathcal{A}_1), \\
\underline{\mathcal{E}}^{\underline{\alpha}} &= e^{-\frac{1}{6}\phi}\,\mathcal{E}^{\underline{\alpha}} + e^{\frac{1}{6}\phi}\,\underline{\mathcal{E}}^{11}\,(\Gamma^{11}\lambda)^{\underline{\alpha}},
\end{aligned} \tag{5.47}
$$

where the index 11 is identified with the index 7 of the U(1) fiber direction of S^7 and

$$
\mathcal{A}_1(x,y,\vartheta,\upsilon) = e^{-\frac{2}{3}\phi(\upsilon)}\,dZ^{\mathcal{M}}\,(E_{\mathcal{M}}{}^a\,\Lambda_a{}^7 + E_{\mathcal{M}}{}^7\,\Lambda_7{}^7)\,. \tag{5.48}
$$

The one forms $\mathcal{E}^{\mathcal{A}}(x,y,\theta) = (\mathcal{E}^A,\,\mathcal{E}^{\underline{\alpha}})$, $\mathcal{A}_1(x,y,\vartheta,\upsilon)$, the spinor superfield $\lambda_{\underline{\alpha}}(x,y,\theta)$, with non-zero components

$$
\lambda_{\alpha i} = -\frac{1}{3}D_{\alpha i}\,\phi(\upsilon)\,, \tag{5.49}
$$

and the scalar superfield $\phi(x,y,\theta)$ do not depend on the 11th coordinate z. They describe, respectively, the supervielbeins, the RR one-form gauge superfield, the dilatino and the dilaton superfields of type IIA supergravity in the *string frame*, eqs. (2.4) and (2.5). The RR field strength F_4 and the NS-NS field strength H_3 given in eqs. (2.6) are obtained from the $D=11$ four-form field strength by the conventional dimensional reduction described in [47]. By construction they solve the type IIA supergravity constraints and describe the $AdS_4 \times CP^3$ background which preserves 24 supersymmetries. The explicit form of these and other relevant IIA superfields has been given in section 2. Using this $AdS_4 \times CP^3$ supergeometry, we can write down the complete Green-Schwarz-type action for the superstring and D-branes on this background (section 3).

6 Conclusion

We have constructed the complete type IIA superspace with 32 fermionic coordinates which describes the $AdS_4 \times CP^3$ vacuum solution of IIA supergravity preserving 24 supersymmetries in terms of superfields depending on 32 fermionic coordinates. Our construction guarantees that the geometry of this superspace and the vacuum configurations of NS-NS and RR superfields living in it solve the type IIA supergravity constraints (and therefore the full set of type IIA equations of motion).[19] An important qualitative difference with previous constructions of supergeometries is that the $AdS_4 \times CP^3$ superspace is not a coset space and that the type IIA $AdS_4 \times CP^3$ superbackground is not maximally supersymmetric.

[19] As an alternative procedure of deriving this supergeometry one might try to directly solve the type IIA supergravity constraints up to the 32-nd order in fermionic variables taking the 24-component $\mathrm{OSp}(6|4)/\mathrm{U}(3)\times\mathrm{SO}(1,3)$ solution as the initial condition.

JHEP03(2009)015

Having the explicit form of the type IIA $AdS_4 \times CP^3$ supergeometry has allowed us to write down the Green-Schwarz-type action for the superstring and D-branes propagating in this background. This provides us with a concrete framework in which to study the most general classical and quantum dynamics of these branes. These actions complete to the full 32-component superspace the string sigma-model actions based on the $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$ supercoset constructed and studied in [27–29, 31].

We have analyzed the integrability of the classical equations of motion of the superstring in different submanifolds of the full $AdS_4 \times CP^3$ superspace. For the submanifold described by the $\mathrm{OSp}(6|4)/\mathrm{U}(3)\times\mathrm{SO}(1,3)$ supercoset, the classical equations of motion are integrable, as already has been shown in [27, 28] following the integrability criteria for sigma-models based on supercosets discovered by Bena, Polchinski and Roiban [32]. We have also considered the supergeometry corresponding to the "complementary" submanifold in $AdS_4 \times CP^3$ superspace. Here we find that this sector of the theory is not based on a supercoset, but on a "twisted" $\mathrm{OSp}(2|4)/\mathrm{SO}(2) \times \mathrm{SO}(1,3)$ superspace, whose supergeometry we have explicitly constructed by restricting the total superspace to this submanifold. Whether the equations of motion in this sector of the theory are classical integrable remains an important open problem. The fact that the complete $AdS_4 \times CP^3$ superspace is not a coset space requires that more general methods are used to prove whether the superstring equations of motion are classically integrable. The explicit construction in this paper of the geometry for the $AdS_4 \times CP^3$ superspace provides a framework in which to study this problem.

Another important question for the future is to understand whether the classical dynamics of the string worldsheet can be encoded in the Hamiltonian describing the spectrum of anomalous dimensions of the holographic dual ABJM theory, extending to this holographic correspondence the analogous results found for the AdS_5/CFT_4 correspondence. It also remains a challenge to find more arguments in favour of the exact integrability of the planar dilatation operator in the ABJM theory. The ultimate fate of the classical integrability of the Green-Schwarz superstring action in $AdS_4 \times CP^3$ and the integrability of the planar ABJM dilatation operator are likely to be related, and remain amongst the most important open problems in this new holographic correspondence.

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A Main notation and conventions

The convention for the ten and eleven dimensional metrics is the 'almost plus' signature $(-,+,\dots,+)$. Generically, the tangent space vector indices are labeled by letters from the beginning of the Latin alphabet, while letters from the middle of the Latin alphabet stand for curved (world) indices. The spinor indices are labeled by Greek letters.

AdS_4 space

AdS_4 is parametrized by the coordinates x^m and its vielbeins are $e^a = dx^m\, e_m{}^a(x)$, $m = 0,1,2,3$; $a = 0,1,2,3$. The $D = 4$ gamma-matrices satisfy:

$$\{\gamma^a, \gamma^b\} = 2\,\eta^{ab}\,, \qquad \eta^{ab} = \mathrm{diag}\,(-,+,+,+)\,, \tag{A.1}$$

$$\gamma^5 = i\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3, \qquad \gamma^5\,\gamma^5 = 1\,. \tag{A.2}$$

The charge conjugation matrix C is antisymmetric, the matrices $(\gamma^a)_{\alpha\beta} \equiv (C\,\gamma^a)_{\alpha\beta}$ and $(\gamma^{ab})_{\alpha\beta} \equiv (C\,\gamma^{ab})_{\alpha\beta}$ are symmetric and $\gamma^5_{\alpha\beta} \equiv (C\gamma^5)_{\alpha\beta}$ is antisymmetric, with $\alpha,\beta = 1,2,3,4$ being the indices of a 4-dimensional spinor representation of SO(1,3) or SO(2,3).

CP^3 space

CP^3 is parametrized by the coordinates $y^{m'}$ and its vielbeins are $e^{a'} = dy^{m'} e_{m'}{}^{a'}(y)$, $m' = 1,\dots,6$; $a' = 1,\dots,6$. The $D = 6$ gamma-matrices satisfy:

$$\{\gamma^{a'}, \gamma^{b'}\} = 2\,\delta^{a'b'}\,, \qquad \delta^{a'b'} = \mathrm{diag}\,(+,+,+,+,+,+)\,, \tag{A.3}$$

$$\gamma^7 = \frac{i}{6!}\,\epsilon_{a'_1a'_2a'_3a'_4a'_5a'_6}\,\gamma^{a'_1}\cdots\gamma^{a'_6} \qquad \gamma^7\,\gamma^7 = 1\,. \tag{A.4}$$

The charge conjugation matrix C' is symmetric and the matrices $(\gamma^{a'})_{\alpha'\beta'} \equiv (C\,\gamma^{a'})_{\alpha'\beta'}$ and $(\gamma^{a'b'})_{\alpha'\beta'} \equiv (C'\,\gamma^{a'b'})_{\alpha'\beta'}$ are antisymmetric, with $\alpha',\beta' = 1,\dots,8$ being the indices of an 8-dimensional spinor representation of SO(6) or SO(8).

Seven-sphere

S^7 is parametrized by the coordinates $\hat{y}^{\hat{m}'} = (y^{m'}, z)$, where z stands for the coordinate of the Hopf fiber in the description of S^7 as a U(1) bundle over CP^3, and its vielbeins are $e^{\hat{a}'} = d\hat{y}^{\hat{m}'}\, e_{\hat{m}'}{}^{\hat{a}'}(\hat{y})$, $\hat{m}' = (m',7)$; $\hat{a}' = (a',7)$. The $D = 7$ gamma-matrices are given by

$$\gamma^{\hat{a}'} = (\gamma^{a'}, \gamma^7)\,, \tag{A.5}$$

and satisfy the Clifford algebra

$$\{\gamma^{\hat{a}'}, \gamma^{\hat{b}'}\} = 2\,\delta^{\hat{a}'\hat{b}'}\,, \qquad \delta^{\hat{a}'\hat{b}'} = \mathrm{diag}\,(+,+,+,+,+,+,+)\,. \tag{A.6}$$

JHEP03(2009)015

Type IIA $AdS_4 \times CP^3$ superspace

The type IIA superspace whose bosonic body is $AdS_4 \times CP^3$ is parametrized by 10 bosonic coordinates $X^M = (x^m, \, y^{m'})$ and 32-fermionic coordinates $\theta^{\underline{\mu}} = (\theta^{\mu\mu'})$ ($\mu = 1,2,3,4$; $\mu' = 1,\ldots,8$). These combine into the superspace supercoordinates $Z^{\mathcal{M}} = (x^m, \, y^{m'}, \, \theta^{\mu\mu'})$. The type IIA supervielbeins are

$$\mathcal{E}^{\mathcal{A}} = dZ^{\mathcal{M}} \, \mathcal{E}_{\mathcal{M}}{}^{\mathcal{A}}(Z) = (\mathcal{E}^{A}, \, \mathcal{E}^{\underline{\alpha}}) \,, \qquad \mathcal{E}^{A}(Z) = (\mathcal{E}^{a}, \, \mathcal{E}^{a'}) \,, \qquad \mathcal{E}^{\underline{\alpha}}(Z) = \mathcal{E}^{\alpha\alpha'} \,. \tag{A.7}$$

The $D = 10$ gamma-matrices Γ^A are given by

$$\begin{aligned} &\{\Gamma^A, \, \Gamma^B\} = 2\eta^{AB}, \qquad \Gamma^A = (\Gamma^a, \, \Gamma^{a'}) \,, \\ &\Gamma^a = \gamma^a \otimes \mathbf{1}, \quad \Gamma^{a'} = \gamma^5 \otimes \gamma^{a'}, \quad \Gamma^{11} = \gamma^5 \otimes \gamma^7, \quad a = 0,1,2,3; \quad a' = 1,\ldots,6 \,. \end{aligned} \tag{A.8}$$

The charge conjugation matrix is $\mathcal{C} = C \otimes C'$.

Torsion constraint

Our convention for the essential constraint on the torsion $D\,\mathcal{E}^{\mathcal{A}} = \frac{1}{2}\,\mathcal{E}^{\mathcal{C}}\,\mathcal{E}^{\mathcal{B}}\,T_{\mathcal{BC}}{}^{\mathcal{A}}$ of IIA supergravity is $T^A_{\underline{\alpha\beta}} = 2\Gamma^A_{\underline{\alpha\beta}}$. This choice is related to the form of the OSp(8|4) algebra (appendix B, eq. (B.7)) and differs from that of [47] by the factor $2i$.

Explicit form of the vielbeins and connections of $\mathrm{OSp}(6|4)/\mathrm{U}(3) \times \mathrm{SO}(1,3)$

The Cartan form is

$$\begin{aligned} K^{-1}_{10,24}\, dK_{10,24} = {} & E^a_{10,24}\, P_a + E^{a'}_{10,24}\, P_{a'} + E^{\alpha a'}_{10,24}\, Q_{\alpha a'} \\ & + \frac{1}{2}\Omega^{ab}_{10,24}\, M_{ab} + \frac{1}{2}\,\Omega^{a'b'}_{10,24}\,(L_{a'b'} - \frac{1}{6}\,J_{a'b'}\,J^{c'd'}\,L_{c'd'}) + A_{10,24}\,T_1. \end{aligned} \tag{A.9}$$

Computing these quantities explicitly using the commutation relations (B.15), the form of the SU(4) generators of appendix C.2 and applying the method described e.g. in [22–25] one finds

$$\begin{aligned} E^a_{10,24} &= e^a(x) - 4\vartheta\gamma^a\, \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}^2_{24}}\, D_{24}\vartheta, \\ E^{a'}_{10,24} &= e^{a'}(y) - 4\vartheta\gamma^{a'}\gamma^5\, \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}^2_{24}}\, D_{24}\vartheta\,, \\ E^{\alpha a'}_{10,24} &= \left(\frac{\sinh \mathcal{M}_{24}}{\mathcal{M}_{24}} D_{24}\vartheta\right)^{\alpha a'}\,, \\ \Omega^{ab}_{10,24} &= \omega^{ab}(x) + 8i\vartheta\gamma^{ab}\gamma^5\, \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}^2_{24}} D_{24}\vartheta\,, \\ \Omega^{a'b'}_{10,24} &= \omega^{a'b'}(y) - 4i\vartheta(\gamma^{a'b'} - iJ^{a'b'}\gamma^7)\gamma^5\, \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}^2_{24}}\, D_{24}\vartheta\,, \\ A_{10,24} &= \frac{1}{8} J_{a'b'}\Omega^{a'b'}_{10,24} = A(y) - 4\,\vartheta\gamma^7\gamma^5\, \frac{\sinh^2 \mathcal{M}_{24}/2}{\mathcal{M}^2_{24}}\, D_{24}\vartheta\,, \end{aligned} \tag{A.10}$$

where

$$(\mathcal{M}^2_{24})^{\alpha a'}{}_{\beta b'} = 4i\vartheta^{\alpha}_{b'}(\vartheta^{a'}\gamma^5)_{\beta} - 4i\delta^{a'}_{b'}\vartheta^{\alpha c'}(\vartheta\gamma^5)_{\beta c'} - 2i(\gamma^5\gamma^a\vartheta)^{\alpha a'}(\vartheta\gamma_a)_{\beta b'} - i(\gamma^{ab}\vartheta)^{\alpha a'}(\vartheta\gamma_{ab}\gamma^5)_{\beta b'}. \tag{A.11}$$

The derivative appearing in the above equations is defined as

$$D_{24}\vartheta = \mathcal{P}_6\left(d + ie^a\gamma^5\gamma_a + ie^{a'}\gamma_{a'} - \frac{1}{4}\omega^{ab}\gamma_{ab} - \frac{1}{4}\omega^{a'b'}\gamma_{a'b'}\right)\vartheta\,, \tag{A.12}$$

where $e^a(x)$, $e^{a'}(y)$, $\omega^{ab}(x)$, $\omega^{a'b'}(y)$ and $A(y)$ are the vielbeins and connections of the bosonic $AdS_4 \times CP^3$ solution (see section 4).

The U(3)-connection $\Omega^{a'b'}_{10,24} = \Omega^{a'b'}_{\mathrm{SU(3)}} + \frac{4}{3}A_{10,24}\,J^{a'b'}$ satisfies the condition

$$(P^-)_{a'b'}{}^{c'd'}\Omega_{c'd'} = \frac{1}{2}(\delta_{[a'}{}^{c'}\,\delta_{b']}{}^{d'} - J_{[a'}{}^{c'}\,J_{b']}{}^{d'})\Omega_{c'd'} = 0\,, \tag{A.13}$$

where $J_{a'b'}$ is the Kähler form on CP^3. Remember also that $\vartheta = \mathcal{P}_6\,\theta$ (see eqs. (C.8) and (C.12)).

Superspace OSp(8|4)/SO(7) × SO(1, 3)

Its bosonic body is $AdS_4 \times S^7$ and it is parametrized by the supercoordinates $\hat{Z}^{\hat{\mathcal{M}}} = (Z^{\mathcal{M}},\, z) = (x^m,\, y^{m'},\, z,\, \theta^{\mu\mu'})$. The corresponding supervielbeins are

$$E^{\hat{\mathcal{A}}}_{11|32} = d\hat{Z}^{\hat{\mathcal{M}}}\,\hat{E}_{\hat{\mathcal{M}}}{}^{\hat{\mathcal{A}}}(\hat{Z}) = dZ^{\mathcal{M}}\,\hat{E}_{\mathcal{M}}{}^{\hat{\mathcal{A}}}(\hat{Z}) + dz\,\hat{E}_7{}^{\hat{\mathcal{A}}}(\hat{Z}) = (\hat{E}^A,\,\hat{E}^7,\,\hat{E}^{\alpha\alpha'})\,. \tag{A.14}$$

The label 7 stands for the 7th direction along S^7 and 11-th direction of $D = 11$.

B OSp(8|4), OSp(2|4) and OSp(6|4)

OSp(8|4) superalgebra[20]

This superalgebra consists of the following:

SO(2, 3) ≃ Sp(4) subalgebra.

$$[P_a, P_b] = -4M_{ab}, \qquad [M_{ab}, M_{cd}] = \eta_{ac}\,M_{bd} + \eta_{bd}\,M_{ac} - \eta_{bc}\,M_{ad} - \eta_{ad}\,M_{bc}\,, \tag{B.1}$$

$$[M_{ab}, P_c] = \eta_{ac}\,P_b - \eta_{bc}\,P_a \tag{B.2}$$

where P_a are the generators of AdS_4 translations and M_{ab} are the generators of SO(1,3).

SO(8) subalgebra.

$$[M_{\tilde{a}'\tilde{b}'},\, M_{\tilde{c}'\tilde{d}'}] = \delta_{\tilde{a}'\tilde{c}'}\,M_{\tilde{b}'\tilde{d}'} - \delta_{\tilde{b}'\tilde{c}'}\,M_{\tilde{a}'\tilde{d}'} + \delta_{\tilde{b}'\tilde{d}'}\,M_{\tilde{a}'\tilde{c}'} - \delta_{\tilde{a}'\tilde{d}'}\,M_{\tilde{b}'\tilde{c}'}\,. \tag{B.3}$$

where

$$M_{\tilde{a}'\tilde{b}'} = (M_{a'b'},\, M_{a'7},\, M_{a'8},\, M_{78})\,, \tag{B.4}$$

and $M_{a'b'}$ $(a', b' = 1, \ldots, 6)$ are the generators of SO(6).

[20]Our conventions are similar to those in [61] modulo the minus sign in the definition of the generators of **SO(1, 3)** and **SO(8)**.

JHEP03(2009)015

Supersymmetry generators $Q_{\alpha\alpha'}$ in OSp(8|4)

$$[P_a,\, Q_{\alpha\alpha'}] = i(Q_{\alpha'}\,\gamma^5\gamma_a)_\alpha\,, \qquad [M_{ab}\,, Q_{\alpha\alpha'}] = -\frac{1}{2}\,(Q_{\alpha'}\,\gamma_{ab})_\alpha\,, \tag{B.5}$$

$$[M_{\tilde{a}'\tilde{b}'}\,, Q_{\alpha\alpha'}] = -\frac{1}{2}\,(Q_\alpha\,\tilde{\gamma}_{\tilde{a}'\tilde{b}'})_{\alpha'}\,, \tag{B.6}$$

$$\{Q_{\alpha\alpha'},\, Q_{\beta\beta'}\} = -2\,C'_{\alpha'\beta'}\,(\gamma^a_{\alpha\beta}\,P_a - i(\gamma^5\gamma^{ab})_{\alpha\beta}\,M_{ab}) - i\gamma^5_{\alpha\beta}\,(\tilde{\gamma}^{\tilde{a}'\tilde{b}'})_{\alpha'\beta'}\,M_{\tilde{a}'\tilde{b}'}, \tag{B.7}$$

where $\alpha = 1,2,3,4$ are Spin(2,3) indices and $\alpha' = 1,\ldots,8$ are Spin(8) indices. We remind the reader that the matrices $C'_{\alpha'\beta'}$, $\gamma^a_{\alpha\beta} = (C\gamma^a)_{\alpha\beta}$ and $\gamma^{ab}_{\alpha\beta} \equiv (C\gamma^{ab})_{\alpha\beta}$ are symmetric in spinor indices and the matrices $C_{\alpha\beta}$, $\gamma^5_{\alpha\beta} \equiv (C\gamma^5)_{\alpha\beta}$ and $(\tilde{\gamma}^{\tilde{a}'\tilde{b}'})_{\alpha'\beta'}$ are antisymmetric. The 8×8 matrices $\tilde{\gamma}^{\tilde{a}'\tilde{b}'}$ — which generate SO(8) — are given by

$$\tilde{\gamma}^{\tilde{a}'\tilde{b}'} = -\tilde{\gamma}^{\tilde{b}'\tilde{a}'} = (\gamma^{a'b'},\, \gamma^{a'7},\, \gamma^{a'8},\, \gamma^{78})\,, \qquad \gamma^{a'8} \equiv i\,\gamma^{a'}\,, \qquad \gamma^{78} \equiv i\,\gamma^7. \tag{B.8}$$

OSp(2|4) **superalgebra**

This algebra has 8 Grassmann-odd generators $\mathcal{Q}_{\alpha i}$ $(i = 1,2)$ which obey the following (anti)commutation relations

$$[P_a,\, \mathcal{Q}_{\alpha i}] = i(\mathcal{Q}_i\,\gamma^5\gamma_a)_\alpha\,, \qquad [M_{ab},\, \mathcal{Q}_{\alpha i}] = -\frac{1}{2}\,(\mathcal{Q}_i\,\gamma_{ab})_\alpha\,, \tag{B.9}$$

$$[T_2, \mathcal{Q}_{\alpha i}] = 2\epsilon_i{}^j\,\mathcal{Q}_{\alpha j}\,, \tag{B.10}$$

$$\{\mathcal{Q}_{\alpha i},\, \mathcal{Q}_{\beta j}\} = -2\,\delta_{ij}\,(\gamma^a_{\alpha\beta}\,P_a - i(\gamma^5\gamma^{ab})_{\alpha\beta}\,M_{ab}) - 2i\gamma^5_{\alpha\beta}\,\epsilon_{ij}\,T_2, \tag{B.11}$$

where P_a and M_{ab} are the generators of SO(2,3) and T_2 is the generator of SO(2) and $\epsilon_{ij} = -\epsilon_{ji}$, $\epsilon_{12} = 1$.

As a subalgebra of OSp(8|4) the superalgebra OSp(2|4) can be obtained from eqs. (B.5)–(B.7) by singling out 8 fermionic generators $\mathcal{Q}_{\alpha i}$ from the 32 generators $Q_{\alpha\alpha'}$ by applying to the latter the projector $\mathcal{P}_2$ which has two non-zero eigenvalues (see appendix C.2 for more details)

$$\mathcal{P}_2 = \frac{1}{8}\,(2+J)\,, \qquad\qquad J = -iJ_{a'b'}\,\gamma^{a'b'}\,\gamma^7\,, \tag{B.12}$$

$$(\mathcal{P}_2\,Q)_{\alpha\alpha'} \qquad \Longleftrightarrow \qquad \mathcal{Q}_{\alpha i}\,, \tag{B.13}$$

where $J_{a'b'}$ are components of the Kähler form on CP^3. Thus, there is the following correspondence between the quantities appearing in (B.5)–(B.8) and in (B.9)–(B.11)

$$T_2 = -\frac{1}{2}(J^{a'b'}M_{a'b'} + 2M_{78}), \qquad (\mathcal{P}_2 C'\mathcal{P}_2)_{\alpha'\beta'} \;\Leftrightarrow\; \delta_{ij}, \qquad (\mathcal{P}_2\gamma^7\mathcal{P}_2)_{\alpha'\beta'} \;\Leftrightarrow\; i\epsilon_{ij}. \tag{B.14}$$

OSp(6|4) **superalgebra**

This algebra has 24 Grassmann-odd generators $Q_{\alpha a'}$ $(a' = 1,\ldots,6)$ which obey the following (anti)commutation relations

$$\begin{aligned}
[P_a,\, Q_{\alpha a'}] &= i(Q_{a'}\gamma^5\gamma_a)_\alpha\,, \quad [M_{ab},\, Q_{\alpha a'}] = -\frac{1}{2}\,(Q_{a'}\,\gamma_{ab})_\alpha\,,\\
[M_{a'b'},\, Q_{\alpha c'}] &= \delta_{a'c'}\,Q_{\alpha b'} - \delta_{b'c'}\,Q_{\alpha a'}\,,\\
\{Q_{\alpha a'}, Q_{\beta b'}\} &= -2\,\delta_{a'b'}\,(\gamma^a_{\alpha\beta}\,P_a - i(\gamma^5\gamma^{ab})_{\alpha\beta}\,M_{ab}) - 4i\,\gamma^5_{\alpha\beta}\,M_{a'b'}\,,
\end{aligned} \tag{B.15}$$

JHEP03(2009)015

where P_a and M_{ab} are the generators of SO(2,3) and $M_{a'b'}$ are the generators of SO(6)

$$[M_{a'\,b'},\, M_{c'\,d'}] = \delta_{a'\,c'}\, M_{b'\,d'} - \delta_{b'\,c'}\, M_{a'\,d'} + \delta_{b'\,d'}\, M_{a'\,c'} - \delta_{a'\,d'}\, M_{b'\,c'}\,. \qquad \text{(B.16)}$$

As a subalgebra of OSp(8|4) the superalgebra OSp(6|4) can be obtained from eqs. (B.5)–(B.7) by singling out 24 fermionic generators $Q_{\alpha a'}$ from the 32 generators $Q_{\alpha\alpha'}$ by applying to the latter the projector $\mathcal{P}_6$ which has six non-zero eigenvalues (see appendix C.2 for more details)

$$\mathcal{P}_6 = \frac{1}{8}\,(6 - J)\,, \qquad J = -iJ_{a'b'}\,\gamma^{a'b'}\,\gamma^7\,, \qquad \text{(B.17)}$$

$$(\mathcal{P}_6\, Q)_{\alpha\alpha'} \qquad \Longleftrightarrow \qquad Q_{\alpha a'}\,. \qquad \text{(B.18)}$$

Thus, there is the following correspondence between the SO(8) generators appearing in (B.5)–(B.8) and the SO(6) generators appearing in (B.15)

$$\frac{1}{4}\,(\mathcal{P}_6\,\tilde\gamma^{\tilde a'\tilde b'}\,\mathcal{P}_6)_{\alpha'\beta'}\, M_{\tilde a'\tilde b'} \quad \Longleftrightarrow \quad M_{a'b'}\,, \qquad (\mathcal{P}_6\, C'\,\mathcal{P}_6)_{\alpha'\beta'} \quad \Longleftrightarrow \quad \delta_{a'b'}. \qquad \text{(B.19)}$$

In particular, the generator T_1 of the U(1) subgroup of the CP^3 structure group, which appeared in sections 4 and 5, is

$$T_1 = \frac{1}{6} J^{a'b'}\, M_{a'b'} - M_{78}\,. \qquad \text{(B.20)}$$

OSp(8|4) closure of $Q_{\alpha a'}$ and $\mathcal{Q}_{\alpha i}$

The anticommutator of $Q_{\alpha a'}$ and $\mathcal{Q}_{\alpha i}$

$$\{Q_{\alpha a'},\, \mathcal{Q}_{\beta i}\} = -4i\,\gamma^5_{\alpha\beta}\, M_{a'i} \qquad \text{(B.21)}$$

produces the generators

$$M_{a'i} = (M_{a'7},\, M_{a'8}) \quad \Longleftrightarrow \quad \frac{1}{4}\,(\mathcal{P}_6\,\tilde\gamma^{\tilde a'\tilde b'}\,\mathcal{P}_2)_{\alpha'\beta'}\, M_{\tilde a'\tilde b'} \qquad \text{(B.22)}$$

that correspond to the coset SO(8)/SO(6)×SO(2) and thus complement the SO(6)×SO(2) generators $M_{a'b'}$ and T_2 (which can be associated with (redefined) M_{78}) to complete the full SO(8) algebra. Finally, the OSp(2|4) and OSp(6|4) superalgebras complete the full OSp(8|4) superalgebra with the following commutation relations

$$[M_{a'i},\, Q_{\alpha b'}] = \delta_{a'b'}\, \mathcal{Q}_{\alpha i}\,, \qquad [M_{a'i},\, \mathcal{Q}_{\alpha j}] = -\delta_{ij}\, Q_{\alpha a'}\,. \qquad \text{(B.23)}$$

C SU(3) × U(1) embeddings into SO(6)

C.1 SU(3) × U(1) embedding into SO(6) and the CP^3 coset generators

Let $M_{a'b'} = -M_{b'a'}$ ($a', b' = 1, \ldots, 6$) be the 15 generators of the SO(6) algebra (B.16).

Let $J_{a'b'} = -J_{b'a'}$ be a constant antisymmetric matrix (determining the components of the Kähler form on CP^3) satisfying the relations

$$J_{a'b'} = -J_{b'a'}\,, \qquad J_{a'c'}\, J^{c'}{}_{b'} = -\delta_{a'b'}\,, \qquad \epsilon_{a'b'c'd'e'f'}\, J^{a'b'}\, J^{c'd'} = 8\, J_{e'f'}. \qquad \text{(C.1)}$$

Let $(P^{\pm})_{a'b'}{}^{c'd'}$ be the following 15×15 projection matrices

$$(P^{\pm})_{a'b'}{}^{c'd'} = \frac{1}{2}\,(\delta_{[a'}{}^{c'}\,\delta_{b']}{}^{d'} \pm J_{[a'}{}^{c'}\,J_{b']}{}^{d'}), \qquad P^+ + P^- = \mathbf{1}\,. \tag{C.2}$$

The matrix P^+ has 9 non-zero eigenvalues and the matrix P^- has 6 non-zero eigenvalues.

Then the generators

$$L_{a'b'} = (P^+)_{a'b'}{}^{c'd'}\,M_{c'd'} \tag{C.3}$$

form the algebra $\mathrm{U}(3) = \mathrm{SU}(3)\times \mathrm{U}(1) \subset \mathrm{SO}(6)$ with $\mathrm{SU}(3)$ generated by

$$L_{a'b'} - \frac{1}{6}\,J_{a'b'}\,J^{c'd'}\,M_{c'd'} \tag{C.4}$$

and the $\mathrm{U}(1)$ generated by

$$T' = -\frac{1}{2}\,J^{c'd'}\,M_{c'd'}\,. \tag{C.5}$$

The remaining generators of $\mathrm{SU}(4)\simeq \mathrm{Spin}(6)$, namely

$$K_{a'b'} = (P^-)_{a'b'}{}^{c'd'}\,M_{c'd'} \tag{C.6}$$

form the coset space $CP^3 = \mathrm{SU}(4)/\mathrm{SU}(3)\times \mathrm{U}(1)$. They have the following generic form of the commutation relations

$$[K,K] = L, \qquad [K,L] = K\,. \tag{C.7}$$

For the construction of the $AdS_4\times CP^3$ superspace we have, however, used a different realization of the SU(4) algebra introduced below.

C.2 $\mathrm{SU}(3)\times\mathrm{U}(1)$ embedding into $\mathrm{Spin}(6)$ and its extension to $\mathrm{SU}(4)$ and $\mathrm{Spin}(8)$ via $\mathrm{Spin}(7)$

The necessity of understanding such an embedding is caused by the fact that the 24 fermionic generators Q of the $\mathrm{OSp}(6|4)$ superalgebra (which is the super-isometry of the $AdS_4\times CP^3$ solution of IIA supergravity preserving 24 supersymmetries) have a natural realization as a direct product of 4-dimensional spinors of $\mathrm{Sp}(4)\simeq \mathrm{Spin}(2,3)$ and 6-dimensional vectors of SO(6), i.e. $Q_{\alpha a'}$ carry the $\mathrm{Spin}(2,3)$ spinor indices $\alpha = 1,2,3,4$ and SO(6) vector indices $a' = 1,\ldots,6$. The structure of the $\mathrm{OSp}(6|4)$ superalgebra is given in eqs. (B.15).

At the same time the fermionic variables $\theta^{\underline{\alpha}}$ of IIA supergravity carry 32-component spinor indices of $\mathrm{Spin}(1,9)$ which in the $AdS_4\times CP^3$ background naturally split into 4-dimensional $\mathrm{Spin}(1,3)$ indices and 8-dimensional spinor indices of Spin(6), i.e. $\theta^{\underline{\alpha}} = \theta^{\alpha\alpha'}$ ($\alpha = 1,2,3,4$; $\alpha' = 1,\ldots,8$). 24 of these θ's should correspond to the unbroken supersymmetries of the $AdS_4\times CP^3$ background generated by the 24 $Q_{\alpha a'}$.

These 24 θ are singled out by a projector introduced in [40] which is constructed using the Kähler form (C.1) and seven 8×8 antisymmetric gamma-matrices (A.3). The 8×8 projector matrix has the following form

$$\mathcal{P}_6 = \frac{1}{8}(6 - J)\,, \tag{C.8}$$

where the 8×8 matrix

$$J = -iJ_{a'b'}\,\gamma^{a'b'}\,\gamma^7 \qquad \text{such that} \qquad J^2 = 4J+12 \tag{C.9}$$

has six eigenvalues -2 and two eigenvalues 6, i.e. its diagonalization results in

$$J = \mathrm{diag}(-2,-2,-2,-2,-2,-2,6,6)\,. \tag{C.10}$$

Therefore, the projector (C.8) when acting on an 8-dimensional spinor annihilates 2 and leaves 6 of its components, while the complementary projector

$$\mathcal{P}_2 = \frac{1}{8}(2+J)\,, \qquad \mathcal{P}_2+\mathcal{P}_6 = \mathbf{1} \tag{C.11}$$

annihilates 6 and leaves 2 spinor components.

Thus the spinor

$$\vartheta^{\alpha\alpha'} = (\mathcal{P}_6\,\theta)^{\alpha\alpha'} \qquad \Longleftrightarrow \qquad \vartheta^{\alpha a'} \qquad a'=1,\ldots,6 \tag{C.12}$$

has 24 non-zero components and the spinor

$$\upsilon^{\alpha\alpha'} = (\mathcal{P}_2\,\theta)^{\alpha\alpha'} \qquad \Longleftrightarrow \qquad \upsilon^{\alpha i} \qquad i=1,2 \tag{C.13}$$

has 8 non-zero components. The latter corresponds to the eight supersymmetries broken by the $AdS_4\times CP^3$ background.

We would like to relate the 24-component fermionic variable $\vartheta^{\alpha a'}$ to the Grassmann-odd generators $Q_{\alpha a'}$ taking values in the 6-dimensional vector representation of Spin(6) $\simeq$ SU(4). To this end, remember that the original fermionic variable $\theta^{\alpha\alpha'}$ takes values in the 8-dimensional spinor representation of Spin(6) $\simeq$ SU(4), generated by the antisymmetric product of 6 gamma-matrices $\gamma^{a'}$

$$M_{a'b'} = -\frac{1}{2}\,\gamma_{a'b'}\,, \qquad \gamma_{a'b'} \equiv \frac{1}{2}\,(\gamma_{a'}\,\gamma_{b'} - \gamma_{b'}\,\gamma_{a'})\,. \tag{C.14}$$

The projected spinor (C.12) will therefore transform by the generators of the form

$$L_{a'b'} = -\frac{1}{2}\,\mathcal{P}_6\,\gamma_{a'b'}\,\mathcal{P}_6\,. \tag{C.15}$$

The question is what algebra is generated by (C.15)? Naively, one might think that it is again Spin(6) $\sim$ SU(4). However, it turns out that only the generators of the U(3) subgroup of Spin(6) survive under the action of the projector $\mathcal{P}_6$. Namely, using the (anti)commutation relation of J (defined in (C.9)) with $\gamma^{a'}$

$$J\,\gamma^{a'} + \gamma^{a'}\,J = -4i\,J^{a'}{}_{b'}\,\gamma^{b'}\,\gamma^7\,, \qquad [\gamma_{a'b'},\,J] = 8i\,J_{[a'}{}^{c'}\,\gamma_{b']c'}\,\gamma^7 \tag{C.16}$$

one can show that the following identities hold

$$L_{a'b'} = -\frac{1}{2}\,\mathcal{P}_6\,\gamma_{a'b'}\,\mathcal{P}_6 = -\frac{1}{2}\,(P^+)_{a'b'}{}^{c'd'}\,\mathcal{P}_6\,\gamma_{c'd'}\,\mathcal{P}_6, \quad (P^-)_{a'b'}{}^{c'd'}\,\mathcal{P}_6\,\gamma_{c'd'}\,\mathcal{P}_6 = 0\,, \tag{C.17}$$

$$\mathcal{P}_6\,\gamma_{a'b'}\,\mathcal{P}_2 = (P^-)_{a'b'}{}^{c'd'}\,\mathcal{P}_6\,\gamma_{c'd'}\,\mathcal{P}_2\,, \qquad (P^+)_{a'b'}{}^{c'd'}\,\mathcal{P}_6\,\gamma_{c'd'}\,\mathcal{P}_2 = 0\,, \tag{C.18}$$

where $P^{\pm}$ were defined in (C.2). Thus, in view of the consideration of Subsection C.1 the operators (C.15) indeed generate the U(3) algebra, their SU(3) and U(1) subalgebras being generated, respectively, by[21]

$$C_{a'b'}{}^{I}\, L_I = 2L_{a'b'} - \frac{i}{3}\, J_{a'b'}\, \mathcal{P}_6\, \gamma^7\, \mathcal{P}_6 \tag{C.19}$$

and

$$T' = \frac{1}{4}\, J_{a'b'}\, \mathcal{P}_6\, \gamma^{a'b'}\, \mathcal{P}_6 = -\frac{i}{2}\, \mathcal{P}_6\, \gamma^7 \mathcal{P}_6 \tag{C.20}$$

(compare eqs. (C.19) and (C.20) with (C.4) and (C.5)).

Note that the CP^3 coset space generators (C.6) do not survive under the $\mathcal{P}_6$ projection. We should therefore find another way to extend the U(3) generators (C.15) to Spin(6) $\simeq$ SU(4). It turns out that the matrices $\mathcal{P}_6\, \gamma_{a'}\, \gamma^7 \mathcal{P}_6$ do this job, i.e. they correspond to the six generators of the coset space $CP^3 = SU(4)/U(3)$. Indeed, using the identities

$$\tilde{P}_{a'} = -\mathcal{P}_6 \gamma_{a'} \gamma^7 \mathcal{P}_6 = -\frac{1}{2}(\delta_{a'}{}^{b'} - iJ_{a'}{}^{b'} \gamma^7)\mathcal{P}_6 \gamma_{b'} \gamma^7 \mathcal{P}_6, \tag{C.21}$$

$$\mathcal{P}_6 \gamma_{a'} \mathcal{P}_2 = \frac{1}{2}(\delta_{a'}{}^{b'} + iJ_{a'}{}^{b'} \gamma^7)\mathcal{P}_6 \gamma_{b'} \mathcal{P}_2, \quad \mathcal{P}_2 \gamma_{a'} \mathcal{P}_6 = \frac{1}{2}(\delta_{a'}{}^{b'} + iJ_{a'}{}^{b'} \gamma^7)\mathcal{P}_2 \gamma_{b'} \mathcal{P}_6 \tag{C.22}$$

and

$$\mathcal{P}_2\, \gamma_{a'}\, \mathcal{P}_2 = 0 \tag{C.23}$$

one can show that $\tilde{P}_{a'}$, defined in (C.21), and the U(3) generators $L_{a'b'}$, defined in eq. (C.15), form the following realization of the Spin(6) $\simeq$ SU(4) algebra

$$[\tilde{P}_{a'}, \tilde{P}_{b'}] = 2L_{a'b'}, \qquad [\tilde{P}_{a'}, L_{b'c'}] = (\delta_{a'b'} - i\, J_{a'b'}\gamma^7)\, \tilde{P}_{c'} - (\delta_{a'c'} - i\, J_{a'c'}\gamma^7)\, \tilde{P}_{b'}\,. \tag{C.24}$$

Note that instead of the generators $\tilde{P}_{a'}$ defined in (C.21) one can equivalently use the generators

$$P_{a'} = J_{a'}{}^{b'}\, \tilde{P}_{b'} = i\, \mathcal{P}_6\, \gamma_{a'} \mathcal{P}_6 \tag{C.25}$$

as the CP^3 translations, as we actually do in the main part of the paper.

The six generators $-\frac{1}{2}\gamma_{a'}\gamma_7$ extend Spin(6) $\simeq$ SU(4) to Spin(7)

$$M_{\hat{a}'\hat{b}'} = (M_{a'b'}, M_{a'7}), \quad M_{a'7} = -M_{7a'} = -\frac{1}{2}\, \gamma_{a'}\gamma_7, \quad \hat{a}' = (a', 7) \tag{C.26}$$

$$[M_{\hat{a}'\,\hat{b}'}, M_{\hat{c}'\,\hat{d}'}] = \delta_{\hat{a}'\,\hat{c}'}\, M_{\hat{b}'\,\hat{d}'} - \delta_{\hat{b}'\,\hat{c}'}\, M_{\hat{a}'\,\hat{d}'} + \delta_{\hat{b}'\,\hat{d}'}\, M_{\hat{a}'\,\hat{c}'} - \delta_{\hat{a}'\,\hat{d}'}\, M_{\hat{b}'\,\hat{c}'}\,. \tag{C.27}$$

Note also that the following matrices further extend the Spin(7) algebra (C.26) to Spin(8)

$$M_{a'8} = -M_{8a'} \equiv -\frac{i}{2}\, \gamma_{a'}\,, \qquad M_{78} \equiv -\frac{i}{2}\gamma_7\,. \tag{C.28}$$

Namely, the Spin(8) algebra is generated by

$$\mathcal{M}_{\tilde{a}'\tilde{b}'} = (M_{a'b'},\, M_{a'7},\, M_{a'8},\, M_{78})\,, \tag{C.29}$$

[21]Note that in the main text, for brevity, the SU(3) generators associated with (C.19) are denoted by L_I (see e.g. eqs. (4.7)–(4.11), (5.15)–(5.16) and (5.22)).

where $M_{a'8}$ and M_{78}, defined in (C.28), correspond to an S^7-sphere coset SO(8)/SO(7).

In terms of the generators $M_{a'7}$ and $M_{a'8}$, the CP^3 generators (C.21) or (C.25) are given by

$$\tilde{P}_{a'} = M_{a'7} + J_{a'}{}^{b'}\, M_{b'8}, \qquad P_{a'} = -M_{a'8} + J_{a'}{}^{b'}\, M_{b'7}.$$

Thus, to reduce 8-component spinors to 6-component "vectors" taking values in the corresponding representation of Spin(6) $\simeq$ SU(4) one should start with the 8-component spinor representations of the Spin(7) algebra (C.26) and apply to them the projector $\mathcal{P}_6$ (C.8).

What about the $\mathcal{P}_2$ projection of $\gamma_{a'b'}$? It has the form similar to eq. (C.17)

$$\frac{1}{2}\,\mathcal{P}_2\,\gamma_{a'b'}\,\mathcal{P}_2 = \frac{1}{2}\,(P^+)_{a'b'}{}^{c'd'}\,\mathcal{P}_2\,\gamma_{c'd'}\,\mathcal{P}_2, \tag{C.30}$$

but now one should remember that $\mathcal{P}_2$ has only 2 non-zero eigenvalues and, hence, the matrix $\mathcal{P}_2\,\gamma_{a'b'}\,\mathcal{P}_2$ is effectively a 2×2 *antisymmetric* matrix (in spinor indices). Since there is only one independent 2×2 antisymmetric matrix, the matrices (C.30) belong to an SO(2) $\simeq$ U(1) algebra which commutes with the SU(4) $\simeq$ SO(6) algebra generated by (C.15) and (C.21).

Thus, the generic form of the matrix (C.30) is $X_{a'b'}\,\epsilon_{ij}$, where $X_{a'b'}$ and ϵ_{ij} is an antisymmetric 6×6 and 2×2 matrix, respectively. Since the only U(3)-invariant antisymmetric 6×6 matrix is $J_{a'b'}$, the matrices (C.30) actually reduce to

$$-\frac{1}{2}\,\mathcal{P}_2\,\gamma_{a'b'}\,\mathcal{P}_2 = -\frac{i}{12}\,J_{a'b'}\,(\mathcal{P}_2\,J\gamma^7\,\mathcal{P}_2) = -\frac{i}{2}\,J_{a'b'}\,(\mathcal{P}_2\,\gamma^7\,\mathcal{P}_2)\,, \tag{C.31}$$

which can also be checked directly using an explicit form of the $\gamma^{a'}$-matrices. The Abelian algebra generated by the 2×2 antisymmetric matrix $-\frac{i}{2}\,\mathcal{P}_2\,\gamma^7\,\mathcal{P}_2$ can be associated with the SO(2) subalgebra of SO(8) which commutes with SO(6) generated by eq. (C.24).

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JHEP03(2009)015

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